Honors precalculus.

Chapter 8 Project. MATRICES

This project needs to be done individually in your own time. You may ask me for clarification but generally it is expected that you will work through the examples and problems on your own. **The project is due the**

n. You may start work on it anytime during our first chapter (Chap 8).

Do this assignment on regular size paper please. Have your name clearly marked on the top of the project with your class period and Honors Precalc/Trig Matrices project as your heading. Number the answers to questions as they appear on the project questions.

You can write in any symbols or draw any diagrams by hand, BUT they must be very neat and precise.

This project score goes into your test category for grading purposes.

Extension of work with matrices.

HAVE YOU EVER WONDERED HOW TO PLAN A NETORK OF ROADS IN A CITY OR TO RANK BASEBALL TEAMS (OR ANY OTHER TEAMS FOR THAT MATTER) ?

This assignment will lead you through some problem solving strategies involving the use of matrices.

Matrices can be used in some practical applications. Specifically they can be used in city planning to offer the best network of 2 way and one way travel between points and to power rank baseball teams.

In this project you will be asked questions after looking through some examples. These tasks will show you how matrices are used in practical problem solving.

A matrix can be used to represent a directed graph. These matrices are often called vertex matrices. A directed graph is a graph that connects points in a certain direction. 3 examples of directed graphs with their corresponding vertex matrices are shown below.

Fig 1:

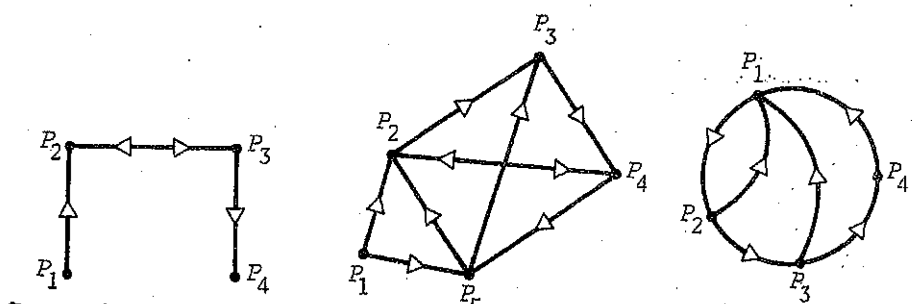


Figure 1 (a) Figure 1. (b) Fig 1 (c)

Figure 2 a)

$$M = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Figure 2 b)

$$M = \begin{bmatrix} 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 \end{bmatrix}$$

Figure 2 c)

$$M = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

These are of the form

$$\begin{array}{cc} & \text{To} \\ \text{From} & \begin{bmatrix} \quad \end{bmatrix} \end{array}$$

By their definition, vertex matrices have the following two properties:

(i) All entries are either 0 or 1.
(ii) All diagonal entries are 0.

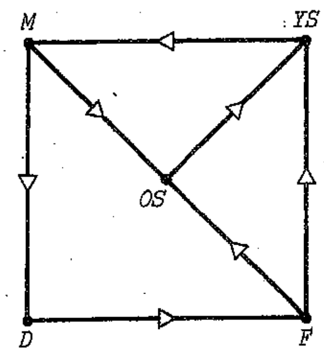
The matrices are formed by entering a 1 in place a_{ij} if the point represented by row i is joined to the point represented by column j in the direction i--->j , otherwise a zero is entered.

Another example of the application of vertex matrices is shown below where family members are "ranked" according to their influence over another family member.

A certain family consists of a mother, father, daughter, and two sons. The family members have influence, or power, over each other in the following ways: the mother can influence the daughter and the oldest son; the father can influence the two sons; the daughter can influence the father; the oldest son can influence the youngest son; and the youngest son can influence the mother. We may model this family influence pattern with a directed graph whose vertices are the five family members. If family member A influences family member B, we write $A \to B$. Figure 3 is the resulting directed graph, where we have used obvious letter designations for the five family members. The vertex matrix of this directed graph is

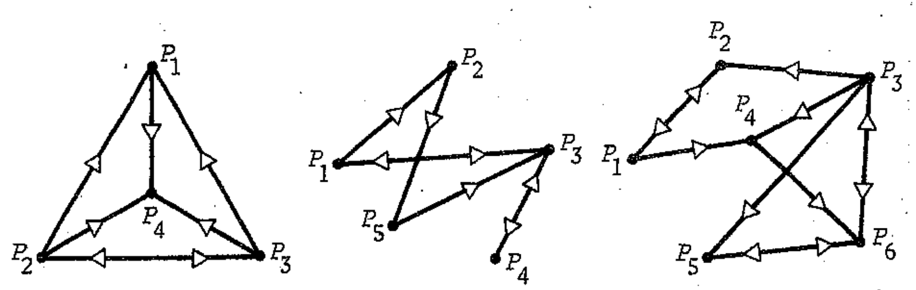
$$\begin{array}{c c} & \begin{array}{ccccc} M & F & D & OS & YS \end{array} \\ \begin{array}{c} M \\ F \\ D \\ OS \\ YS \end{array} & \begin{bmatrix} 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

Figure 3



Using the above examples do the following:

1. Construct the vertex matrix for each of the directed graphs illustrated in Fig. 4



(a) (b) (c)

Figure 4

2. Draw a diagram of the directed graph corresponding to each of the following vertex matrices.

(a) $\begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \end{bmatrix}$ (c) $\begin{bmatrix} 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 \end{bmatrix}$

The number of 2 and 3 step connections between one vertex and another can be found by taking the vertex matrix and squaring or cubing it respectively. Each entry a_{ij} represents the number of 2 and 3 step connections between vertex P_i and P_j. In the following question use your graphing calculator to enter the vertex matrix for the directed graph and then use the square or cube function on your calculator where appropriate.

Let M be the following vertex matrix of a directed graph:

Figure 5

$$\begin{array}{c} \\ P_1 \\ P_2 \\ P_3 \\ P_4 \end{array} \begin{array}{c} \begin{array}{cccc} P_1 & P_2 & P_3 & P_4 \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix} \end{array}.$$

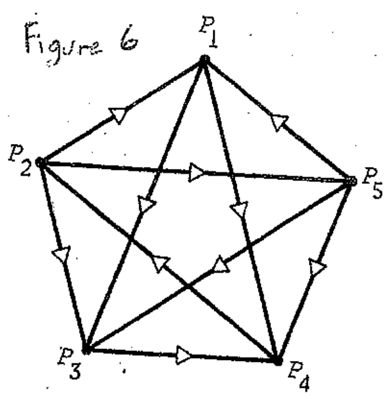
3. a) Draw a diagram of the directed graph for the vertex matrix above.

b) Find the number of 1, 2, and 3 step connections from P_1 to P_2 using the square and cube of the vertex matrix. List these connections. (for example a 2 step connection from P_1 to P_2 could be written as $P_1 \to P_3 \to P_2$)

c) Repeat part b) for connections between P_1 and P_4.

A dominance directed vertex matrix is one in which there are no 2 way connections between vertices. They are all one way connections as shown in the figure below. The corresponding vertex matrix is also shown.

Figure 6



$$M = \begin{bmatrix} 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 \end{bmatrix}.$$

Suppose these vertices represented 5 baseball teams that played each other once. We can use the dominance directed vertex matrix and the square of this matrix to rank these 5 teams. We add the matrix to its square and the resulting matrix represents the total number of 1 and 2 step connections from one vertex to another. The vertex with the largest row sum will be the "powerful vertex" and represent rank 1. The other rankings can be determined by adding their rows and putting them in order. This process is shown below.

Figure 7

$$A = M + M^2 = \begin{bmatrix} 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 2 & 3 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 2 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 1 & 2 & 0 \\ 2 & 0 & 3 & 3 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 2 & 3 & 0 \end{bmatrix}.$$

The row sums of A are:

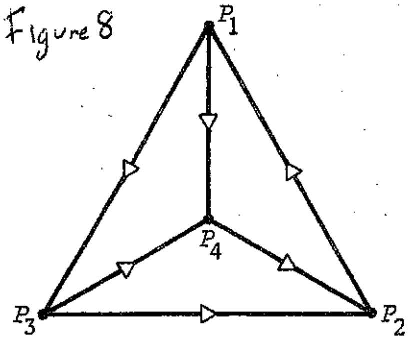
1st row sum = 4
2nd row sum = 9
3rd row sum = 2
4th row sum = 4
5th row sum = 7.

This means that P_2 is ranked highest because it is the most powerful matrix. P_5 would be ranked 2nd because it is the next most powerful matrix, and so on.

Using the above method do the following 2 questions.

4. For the dominance directed graph shown in Fig. 8, construct the vertex matrix and find the "power" of each vertex using the method shown above. Rank these vertices in order from most powerful to least powerful.

Figure 8



5. Five baseball teams play each other one time with the following results:

A beats B, C, D

B beats C, E

C beats D, E

D beats B

E beats A, D

Rank these 5 teams.

Pre Calc Ch 8 Project

ANSWERS

Name ______________

Period ________

1.

a)

b)

c)

2.

a)

b)

c)

3.

a)

b) $M^2 =$

$M^3 =$

# of 1 step connections from P_1 to P_2 = ____

# of 2 step connections from P_1 to P_2 = ____

# of 3 step connections from P_1 to P_2 = ____

$\Longrightarrow$

3 b) continued.

List of 1 step connections from P_1 to P_2

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List of 2 step connections from P_1 to P_2

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List of 3 step connections from P_1 to P_2

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3. c) # 1 step connections from P_1 to P_4 = ______ # 2 step connections from P_1 to P_4 = ______ # 3 step connections from P_1 to P_4 = ______

List of 1 step connections from P_1 to P_4

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List of 2 step connections from P_1 to P_4

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List of 3 step connections from P_1 to P_4

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4. $M =$ $M^2 =$ $M + M^2 =$

1st row sum = ________

2nd row sum = ________

3rd row sum = ________

4th row sum = ________

<u>RANK is:</u>

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5. <u>Directed graph</u> $M =$ $M^2 =$ $M + M^2 =$

↓

RANK

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