Chapter 10 Topics in Analytic Geometry

Course/Section
Lesson Number
Date

Section 10.3 Ellipses

Section Objectives: Students will know how to write the standard form of the equation of an ellipse and how to find the eccentricity of an ellipse.

I. Introduction (pp. 744−747) Pace: 20 minutes

- Define an **ellipse** to be the set of all points (x, y) in a plane the sum of whose distances from two distinct fixed points (**foci**) is constant.

- Draw an ellipse, then define and label the following parts. The midpoint between the foci is the **center**. The line segment through the foci, with endpoints on the ellipses, is the **major axis**. The endpoints of the major axis are the **vertices** of the ellipse. The line segment through the center and perpendicular to the major axis, with endpoints on the ellipse, is the **minor axis**. The endpoints of the minor axis are the **covertices** of the ellipse.

- State that the **standard form of the equation of an ellipse** centered at (h, k) with a horizontal major axis of length $2a$, minor axis of length $2b$, vertices at $(h \pm a, k)$, covertices at $(h, k \pm b)$, and foci at $(h \pm c, 0)$ is $\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$. The standard form of the equation of an ellipse centered at (h, k) with a vertical major axis of length $2a$, minor axis of length $2b$, vertices at $(h, k \pm a)$, covertices at $(h \pm b, k)$, and foci at $(h, k \pm c)$ is $\frac{(y-k)^2}{a^2}+\frac{(x-h)^2}{b^2}=1$. In both cases, $a > b$ and $c^2 = a^2 - b^2$.

- State that a is the distance from the center to the vertices, b is the distance from the center to the covertices, and c is the distance from the center to the foci.

Example 1. Find the center, vertices, and foci of the ellipse given by $9x^2 + 4y^2 = 36$.
$\frac{x^2}{4}+\frac{y^2}{9}=1 \Rightarrow a = 3$, $b = 2$, and $c^2 = 9 - 4 \Rightarrow c = \sqrt{5}$. So, the center is $(0, 0)$, the vertices are at $(0, \pm 3)$, and the foci are at $\left(0, \pm\sqrt{5}\right)$.

Example 2. Find the standard form of the equation of the ellipse centered at the origin with major axis of length 10 and foci at $(\pm 3, 0)$.
We know that $a = 5$ and $c = 3$. Next, $b^2 = 5^2 - 3^2$, or $b = 4$. So the equation is $\frac{x^2}{25}+\frac{y^2}{16}=1$.

Example 3. Sketch the graph of the following ellipse.

$$25x^2 + 9y^2 - 200x + 36y + 211 = 0$$

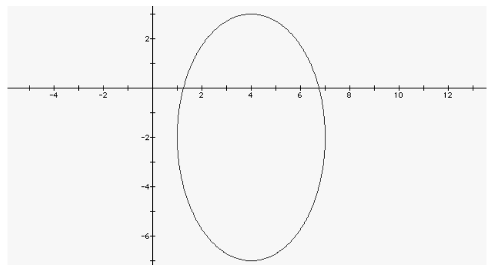
$$25\left(x^2 - 8x\right) + 9\left(y^2 + 4y\right) = -211$$

$$25\left(x^2 - 8x + 16\right) + 9\left(y^2 + 4y + 4\right) = -211 + 400 + 36$$

$$25(x-4)^2 + 9(y+2)^2 = 225$$

$$\frac{(x-4)^2}{9} + \frac{(y+2)^2}{25} = 1$$

So, the center is at (4, −2), $a = 5$, and $b = 3$. To plot the vertices, go up and down 5 units from the center. To plot the coverrtices, go right and left 3 units from the center.



Tip: Discuss the *Technology* feature on page 747 of the text.

II. Application (p. 748) Pace: 5 minutes

Example 4. A passageway in a house is to have straight sides and a semielliptically-arched top. The straight sides are 5 feet tall and the passageway is 7 feet tall at its center and 6 feet wide. Where should the foci be located to make the template for the arch?

$2a = 6 \Rightarrow a = 3$. $b = 7 - 5 = 2$. $c^2 = 3^2 - 2^2 \Rightarrow c = \sqrt{5} \approx 2.236$.

The foci should be about 2.236 feet right and left of the semiellipse.

III. Eccentricity (pp. 748−749) Pace: 5 minutes

- State that the **eccentricity** of an ellipse is $e = c/a$, where $0 < e < 1$.
- State that the eccentricity is a measure of the "ovalness" of the ellipse. If the eccentricity is close to 0, then the ellipse is close to being circular. If the eccentricity is close to 1, then the ellipse is flatter.
- Discuss the *Writing About Mathematics* on page 749 of the text.