Chapter 10 Topics in Analytic Geometry

Course/Section
Lesson Number
Date

Section 10.4 Hyperbolas

Section Objectives: Students will know how to write the standard form of the equation of a hyperbola, how to find the asymptotes of a hyperbola, and how to classify a conic from its general equation.

I. Introduction (pp. 753−754) Pace: 10 minutes

- Define a **hyperbola** to be the set of all points (x, y) in a plane the difference of whose distances from two distinct fixed points (**foci**) is constant.

- Draw a hyperbola, then define and label the following parts. The midpoint between the foci is the **center**. The points at which the line segment through the foci meets the hyperbola are the **vertices**. The line segment joining the vertices is the **transverse axis**.

- State that the **standard form of the equation of a hyperbola** centered at (h, k) with a horizontal transverse axis of length $2a$, vertices at $(h \pm a, k)$, and foci at $(h \pm c, k)$ is $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$. The standard form of the equation of a hyperbola centered at (h, k) with a vertical transverse axis of length $2a$, vertices at $(h, k \pm a)$, and foci at $(h, k \pm c)$ is $\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$. In both cases, $c^2 = a^2 + b^2$.

- State that a is the distance from the center to the vertices, and c is the distance from the center to the foci.

II. Asymptotes of a Hyperbola (pp. 755−757) Pace: 15 minutes

- State that each hyperbola has two asymptotes that intersect at the center of the hyperbola.

- State that for a hyperbola with a horizontal transverse axis, the equations of the asymptotes are $y = k \pm (b/a)(x - h)$, and for a hyperbola with a vertical transverse axis, the equations of the asymptotes are $y = k \pm (a/b)(x - h)$.

- State that for a hyperbola, the **eccentricity** is $e = c/a > 1$. Also, the larger the eccentricity is, the closer the branches of the hyperbola are to being lines.

Example 1. Find the center, vertices, asymptotes, and foci of the hyperbola given by $4y^2 - 9x^2 = 36$.
$\frac{y^2}{9} - \frac{x^2}{4} = 1 \Rightarrow a = 3$, $b = 2$, and $c^2 = 9 + 4 \Rightarrow c = \sqrt{13}$. So, the center is $(0, 0)$, the vertices are at $(0, \pm 3)$, the asymptotes are $y = \pm(3/2)x$, and the foci are at $(0, \pm\sqrt{13})$.

Example 2. Find the standard form of the equation of the hyperbola centered at the origin with transverse axis of length 4 and foci at $(\pm 3, 0)$.
We know that $a = 2$ and $c = 3$. Next $b^2 = 3^2 - 2^2$ or $b = \pm\sqrt{5}$. So the equation is $\frac{x^2}{4} - \frac{y^2}{5} = 1$.

Example 3. Sketch the graph of the hyperbola given by

$$4x^2 - 9y^2 - 24x - 72y - 72 = 0$$

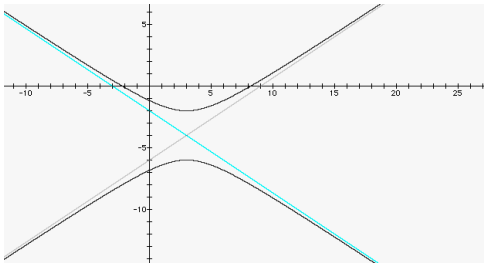
$$4(x^2 - 6x) - 9(y^2 + 8y) = 72$$

$$4(x^2 - 6x + 9) - 9(y^2 + 8y + 16) = 72 + 36 - 144$$

$$4(x-3)^2 - 9(y+4)^2 = -36$$

$$\frac{(y+4)^2}{4} - \frac{(x-3)^2}{9} = 1$$

So, the center is at (3, −4), $a = 2$, and $b = 3$. Go up and down 2 units and right and left 3 units from the center. Use these four points to form the box and draw the asymptotes.



III. Applications (p. 758) Pace: 5 minutes

Example 4. There is a listening station located at $A(2200, 0)$ (in feet) and another at $B(-2200, 0)$. An explosion is heard at station A one second before it is heard at station B. Where was the explosion located?

The explosion occurred on the right branch of a hyperbola with $c = 2200$. Because sound travels at approximately 1100 feet per second, $2a = 1100 \Rightarrow a = 550$. Together these values yield $b^2 = 2200^2 - 550^2 = 4{,}537{,}500$. So, the equation of the hyperbola is $x^2/302{,}500 - y^2/4{,}537{,}500 = 1$.

IV. General Equations of Conics (p. 759) Pace: 5 minutes

- State that the graph of $Ax^2 + Cy^2 + Dx + Ey + F = 0$ is:
  1. a circle if $A = C$,
  2. a parabola if $AC = 0$,
  3. an ellipse if $AC > 0$, or
  4. a hyperbola if $AC < 0$.

Example 5. Classify each of the following.

a) $4x^2 + 5y^2 - 9x + 8y = 0$ ellipse ($AC = 20$)
b) $2x^2 - 5x + 7y - 8 = 0$ parabola ($AC = 0$)
c) $7x^2 + 7y^2 - 9x + 8y - 16 = 0$ circle ($A = C$)
d) $4x^2 - 5y^2 - x + 8y + 1 = 0$ hyperbola ($AC = -20$)

- Assign the *Writing About Mathematics* on page 759 of the text.