Chapter 10 Topics in Analytic Geometry

Course/Section
Lesson Number
Date

Section 10.6 Parametric Equations

Section Objectives: Students will know how to evaluate a set of parametric equations, sketch the curve represented by a set of parametric equations, rewrite a set of parametric equations as a rectangular equation, and find a set of parametric equations for a graph.

I. Plane Curves (p. 771) Pace: 5 minutes

- State the definition of a **plane curve** by stating that if f and g are continuous functions of t on an interval I, then the set of all ordered pairs $(f(t), g(t))$ is a plane curve C. The equations $x = f(t)$ and $y = g(t)$ are **parametric equations** for C, and t is the **parameter.**

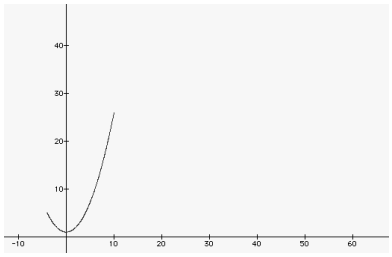
- State that one of the advantages of representing curves by parametric equations rather than by one equation in two variables is that we can represent all the conics by functions.

II. Sketching a Plane Curve (p. 772) Pace: 10 minutes

- One way of sketching plane curves is by point plotting. The advantage of this method is that we can see how the curve is traced in the order of increasing values of t, called the **orientation** of the curve.

Example 1. Sketch the graph of the curve given by $x = 2t$ and $y = t^2 + 1$ on [2, 5].
Make a table of values and plot the points.

t	-2	-1	0	1	2	3	4	5
x	-4	-2	0	2	4	6	8	10
y	5	2	1	2	5	10	17	26



III. Eliminating the Parameter (p. 773−774) Pace: 5 minutes

- State that sometimes we need to look at a single two-variable equation of a curve instead of the parametric equations. To do this we eliminate the parameter by solving one equation for t and substituting into the other equation.

Example 2. Find an equation in x and y for the curve given by

$$x = \frac{1}{t+1}$$

$$y = \frac{3t^2 + 6t + 4}{t^2 + 2t + 1}$$

Solve the first equation for t to obtain $t = \frac{1-x}{x}$.

Substitute this into the second equation and simplify.

$$y = \frac{3\left(\frac{1-x}{x}\right)^2 + 6\left(\frac{1-x}{x}\right) + 4}{\left(\frac{1-x}{x}\right)^2 + 2\left(\frac{1-x}{x}\right) + 1} = \frac{3(1 - 2x + x^2) + 6x(1-x) + 4x^2}{(1 - 2x + x^2) + 2x(1-x) + x^2}$$

$$= \frac{x^2 + 3}{1} = x^2 + 3$$

IV. Finding Parametric Equations for a Graph (pp. 774−775)
Pace: 5 minutes

- State that we now go in the other direction; that is, we are given the equation in x and y and we find a parametric equation for the same curve.

Example 3. Find a set of parametric equations to represent the graph given by $y = x^2 + 1$ using the following parameters.

a) $t = x$
$y = x^2 + 1$
$y = t^2 + 1$
So, the equations are $x = t$ and $y = t^2 + 1$.

b) $t = x + 2$
$y = x^2 + 1$
$y = (t - 2)^2 + 1$
$y = t^2 - 4t + 4 + 1$
$y = t^2 - 4t + 5$
So, the equations are $x = t - 2$ and $y = t^2 - 4t + 5$.