Chapter 10 Topics in Analytic Geometry

Course/Section
Lesson Number
Date

Section 10.7 Polar Coordinates

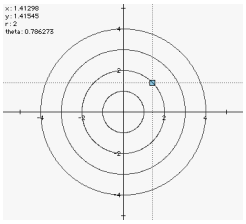
Section Objectives: Students will know how to convert points from rectangular to polar form, and how to write equations in polar form.

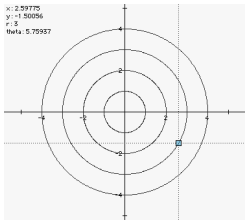
I. Introduction (pp. 779−780) Pace: 10 minutes

- State that the **polar coordinate system** consists of a fixed point O, called the **pole**, and a ray, called the **polar axis**, with its initial point at O.

- State that each point P in the plane can be labeled with **polar coordinates** (r, θ), where r is the directed distance from O to P and θ is an angle in standard position with terminal side at $\overrightarrow{OP}$.

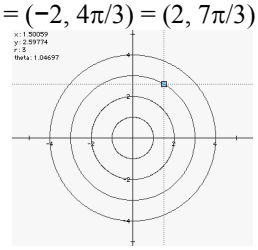
Example 1. Plot the following points in the polar coordinate system.
a) $(2, \pi/4)$ **b)** $(3, -\pi/6)$





- State that the polar representation of a point is not unique. In general, $(r, \theta) = (r, \theta \pm 2\pi n)$ or $(r, \theta) = (-r, \theta \pm (2n - 1)\pi)$.

Example 2. Find three other polar representations for the point $(2, \pi/3)$.
$(2, \pi/3) = (-2, 4\pi/3) = (2, 7\pi/3) = (-2, -2\pi/3)$



II. Coordinate Conversion (pp. 780−781) Pace: 10 minutes

- State that to covert from polar to rectangular and back, we use the following facts: $r^2 = x^2 + y^2$, $\tan\theta = y/x$, $\sin\theta = y/r$, and $\cos\theta = x/r$.

Example 3. Convert the following polar coordinates to rectangular coordinates.

a) $(4, \pi/6)$
$x = 4\cos\pi/6 = 2\sqrt{3}$
$y = 4\sin\pi/6 = 2$
$(4, \pi/6) = (2\sqrt{3}, 2)$

b) $(-2, 5\pi/4)$

$$x = -2\cos\frac{5\pi}{4} = \sqrt{2}$$

$$y = -2\sin\frac{5\pi}{4} = \sqrt{2}$$

$$\left(-2, \frac{5\pi}{4}\right) = \left(\sqrt{2}, \sqrt{2}\right)$$

Example 4. Convert the following rectangular coordinates to polar coordinates.

a) $(-1, -1)$

Since θ is in the third quadrant,

$$\tan\theta = \frac{-1}{-1} = 1 \Rightarrow \theta = \frac{5\pi}{4}$$

$$r = \sqrt{(-1)^2 + (-1)^2} = \sqrt{2}$$

$$(-1, -1) = \left(\sqrt{2}, \frac{5\pi}{4}\right)$$

b) $(0, -2)$

This points lies 2 units down on the y-axis. So, $(0, -2) = (2, 3\pi/2)$.

III. Equation Conversion (p. 782) Pace: 10 minutes

- State that to convert rectangular equations to polar equations, simply replace x with $r\cos\theta$ and y with $r\sin\theta$. Converting polar equations to rectangular equations requires considerable ingenuity.

Example 5. Convert the following polar equations to rectangular equations.

a) $r = 5$

$$r^2 = 25 \Rightarrow x^2 + y^2 = 25$$

b) $\theta = \pi/4$

$\tan\theta = \tan\pi/4 = 1 \Rightarrow y/x = 1 \Rightarrow y = x$

c)

$$r = \frac{1}{1 - \sin 2\theta}$$

$$r - r\sin 2\theta = 1$$

$$r^2 - r^2\sin 2\theta = 1$$

$$r^2 - 2r^2\sin\theta\cos\theta = 1$$

$$r^2 - 2(r\cos\theta)(r\sin\theta) = 1$$

$$x^2 + y^2 - 2xy = 1$$