Chapter 10 Topics in Analytic Geometry

Section 10.8 Graphs of Polar Equations

Course/Section
Lesson Number
Date

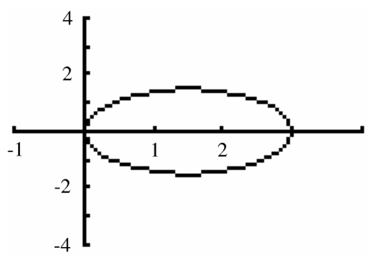
Section Objectives: Students will know how to graph polar equations.

I. Introduction (p. 785) Pace: 5 minutes

- State that we will start by using point plotting.

Example 1. Sketch the graph of $r = 3\cos\theta$.

θ	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{3\pi}{2}$	$\frac{11\pi}{6}$	2π
r	$\frac{3\sqrt{3}}{2}$	$\frac{3\sqrt{2}}{2}$	$\frac{3}{2}$	0	$-\frac{3}{2}$	$-\frac{3\sqrt{3}}{2}$	-3	$-\frac{3\sqrt{3}}{2}$	0	$\frac{3\sqrt{3}}{2}$	3

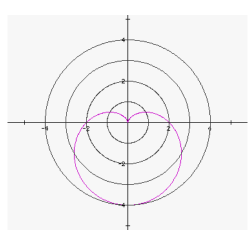


II. Symmetry (pp. 786−787) Pace: 10 minutes

- State that the graph of a polar equation is symmetric with respect to the following if the given substitution yields an equivalent equation.
  1. The line $\theta = \pi/2$: Replace (r, θ) by $(r, \pi - \theta)$ or $(-r, -\theta)$.
  2. The polar axis: Replace (r, θ) by $(r, -\theta)$ or $(-r, \pi - \theta)$.
  3. The pole: Replace (r, θ) by $(r, \pi + \theta)$ or $(-r, \theta)$.

Example 2. Sketch the graph of $r = 2 - 2\sin\theta$.
Replace (r, θ) by $(r, \pi - \theta)$ and see that $r = 2 - 2\sin(\pi - \theta) = 2 - 2\sin\theta$.
Therefore, the graph is symmetric with respect to the line $\theta = \pi/2$.

r	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{7\pi}{4}$	2π
θ	2	1	$2-\sqrt{2}$	0	4	$2+\sqrt{3}$	$2+\sqrt{2}$	2



- State that the three tests for symmetry are sufficient to guarantee symmetry, but not necessary. That is, the tests can all fail to indicate symmetry, but the graph will still have one of the types of symmetry.

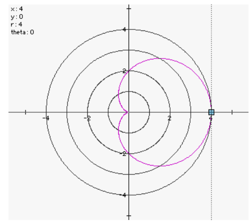
- State that there is another **quick test for symmetry**:
  1. The graph of $r = f(\sin\theta)$ is symmetric with respect to the line $\theta = \pi/2$.
  2. The graph of $r = g(\cos\theta)$ is symmetric with respect to the polar axis.

Copyright © Houghton Mifflin Company. All rights reserved.

III. Zeros and Maximum *r*-Values (pp. 787−788) Pace: 10 minutes

- State that we now consider two other graphing aids: points at which $|r|$ is maximum and points at which $r = 0$.

Example 3. Find the maximum value of $|r|$ for the graph of $r = 2 + 2\cos\theta$.
From the equation, the maximum value of $|r|$ is 4 (at $\theta = 0$).

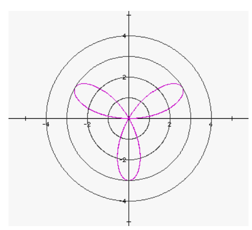


Example 4. Find the point(s) at which $r = 0$ for the graph of $r = 2\sin\theta$.
$2\sin\theta = 0$
$\sin\theta = 0$
$\theta = 0, \pi, 2\pi$

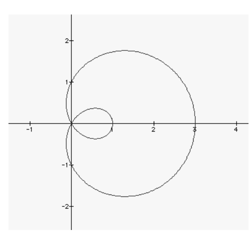
IV. Special Polar Graphs (pp. 789−790) Pace: 10 minutes

- Discuss the graphs on page 789 of the text.

Example 5. Sketch the graph of $r = 3\sin 3\theta$.
This is a rose curve with 3 petals, symmetry with respect to the line $\theta = \pi/2$, maximum values of $|r| = 3$ at $\theta = \pi/6, \pi/2, 5\pi/6$, and $r = 0$ at $\theta = 0, \pi/3, 2\pi/3, \pi$.



Example 6. Sketch the graph of $r = 1 + 2\cos\theta$.
This is a limacon with a loop, symmetry with respect to the polar axis, a maximum value of $|r| = 3$ at $\theta = 0$, and $r = 0$ at $\theta = 2\pi/3, 4\pi/3$.



Larson/Hostetler *Precalculus with Limits* Instructor Success Organizer
Copyright © Houghton Mifflin Company. All rights reserved.