Chapter 10 Topics in Analytic Geometry

Course/Section
Lesson Number
Date

Section 10.9 Polar Equations of Conics

Section Objectives: Students will know how to define conics in terms of eccentricity and how to write equations of conics in polar form.

I. Alternative Definition of Conic (p. 793) Pace: 5 minutes

- State that in some applications it is more convenient to have one of the foci at the origin. For example, if we consider the earth's orbit around the sun, the sun is at one of the foci of an ellipse.

- State the following **alternative definition of a conic**.
  The locus of a point in the plane that moves so that its distance from a fixed point (focus) is in a constant ratio (**eccentricity**) to its distance from a fixed line (directrix) is a **conic**. The eccentricity is denoted by *e*. Furthermore, the conic is
  1. an **ellipse** if $e < 1$,
  2. a **parabola** if $e = 1$, and
  3. a **hyperbola** if $e > 1$.

II. Polar Equations of Conics (pp. 793−795) Pace: 15 minutes

- State that the graph of a polar equation of the form
  $r = \dfrac{ep}{1 \pm e\cos\theta}$, or $r = \dfrac{ep}{1 \pm e\sin\theta}$
  is a conic, where $e > 0$ is the eccentricity and $|p|$ is the distance between the focus and the directrix. Furthermore, equations involving cosine have vertical directrices and equations involving sine have horizontal directrices.

  Example 1. Determine the type of conic given by
  $$r = \frac{12}{3 - 4\sin\theta}$$
  $$r = \frac{4}{1 - (4/3)\sin\theta}$$
  Since $e = 4/3 > 1$, this conic is a hyperbola.

  Example 2. Find the center, vertices and foci of the ellipse given by
  $$r = \frac{15}{5 + 3\cos\theta}.$$
  The vertices are at $(0, 15/8)$ and $(\pi, 15/2)$. Hence the center is at $(45/16, \pi)$. This gives $2a = 75/8 \Rightarrow a = 75/16$. Since $e = 3/5 = c/a \Rightarrow c = 45/16$. Hence the foci are at $(45/8, \pi)$ and the pole.

- State the following facts about the location of the directrix.
  1. Horizontal directrix above the pole: $r = \dfrac{ep}{1 + e\sin\theta}$
  2. Horizontal directrix below the pole: $r = \dfrac{ep}{1 - e\sin\theta}$
  3. Vertical directrix to the right of the pole: $r = \dfrac{ep}{1 + e\cos\theta}$
  4. Vertical directrix to the left of the pole: $r = \dfrac{ep}{1 - e\cos\theta}$

Example 3. Find the polar equation of the parabola with focus at the pole and directrix at $x = -4$.

Because the directrix is vertical and to the left of the pole, the equation is of the form $r = \dfrac{ep}{1 - e\cos\theta}$. Since $e = 1$ and $p = 4$, the equation is $r = \dfrac{4}{1 - \cos\theta}$.

III. Applications (p. 796) Pace: 5 minutes

Example 4. The eccentricity of Mars is approximately 0.09 and its closest distance to the sun is approximately 128,000,000 miles. Find an equation of its orbit around the sun.

We will use an equation of the form $r = \dfrac{ep}{1 + e\cos\theta}$ (you can use any form). Since $e \approx 0.09$, we have $r = 0.09p/(1 + 0.09\cos\theta)$. Since the minimum distance to one focus is 128,000,000, Mars must pass through the point (128,000,000, 0). Use this to find $p \approx 1{,}550{,}222{,}222$. Hence the equation is $r = \dfrac{139{,}520{,}000}{1 + 0.09\cos\theta}$.