Chapter 11 Analytic Geometry in Three Dimensions

Course/Section Lesson Number Date

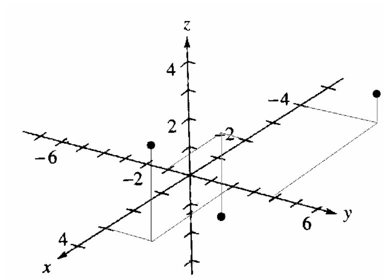
Section 11.1 The Three-Dimensional Coordinate System

Section Objectives: Students will know how to plot points in space, find distances between points in space, and find midpoints of line segments connecting points in space.

I. The Three-Dimensional Coordinate System (pp. 812−813)
Pace: 5 minutes

- State that we now pass a third number line called the ***z*-axis** through the *xy*-plane, normal to the *xy*-plane, so that the origin of this new number line meets the origin of the *xy*-plane. This new coordinate system is called the **three-dimensional coordinate system**. Points in this system are represented by **ordered triples** (x, y, z). Taken in pairs, the axes form three **coordinate planes:** the ***xy*-plane**, the ***xz*-plane**, and the ***yz*-plane**. These three coordinate planes divide the three-dimensional coordinate system into **octants**. The first octant is the one in which all three coordinates are positive. Octants II, III, and IV are found by rotating your right hand counterclockwise around the positive *z*-axis. Octant V is directly below Octant I. Octants VI, VII, and VIII are found by rotating counterclockwise around the negative *z*-axis.

Example 1. Plot the following points.
a) (3, 2, 3) **b)** (-2, -1, -3) **c)** (-4, 4, 1)



II. The Distance and Midpoint Formulas (pp. 813−814) Pace: 10 minutes

- State that the distance between two points in space (x_1, y_1, z_1) and (x_2, y_2, z_2) is $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$.

Example 2. Find the distance between (2, 4, −1) and (5, 6, 2).

$$d = \sqrt{(5-2)^2 + (6-4)^2 + (2-(-1))^2}$$
$$= \sqrt{22}$$

- State that the midpoint of the line segment joining the points (x_1, y_1, z_1) and (x_2, y_2, z_2) is $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2}\right)$.

Example 3. Find the midpoint of the line segment joining the points (2, −3, 1) and (6, 0, 3).

$$\left(\frac{2+6}{2}, \frac{-3+0}{2}, \frac{1+3}{2}\right) = \left(4, -\frac{3}{2}, 2\right)$$

III. The Equation of a Sphere (pp. 814–816) Pace: 10 minutes

- State that a **sphere** with center at (h, k, j) and radius r is the set of all points (x, y, z) whose distance from (h, k, j) is r. Applying the Distance Formula leads to the following **standard form of the equation of a sphere**: $(x - h)^2 + (y - k)^2 + (z - j)^2 = r^2$.

 Example 4. Find the standard form of the equation of the sphere with center at (2, 3, −1) and radius 5.
 $(x - 2)^2 + (y - 3)^2 + (z - (-1))^2 = 5^2$

 Example 5. Find the center and radius of the sphere given by
 $x^2 + y^2 + z^2 - 2x + 6y - 4z - 2 = 0$.
 Completing the square yields
 $(x^2 - 2x + 1) + (y^2 + 6y + 9) + (z^2 - 4z + 4) = 2 + 1 + 9 + 4$
 $(x - 1)^2 + (y + 3)^2 + (z - 2)^2 = 16$.
 Hence, the center is at (1, –3, 2) and the radius is 4.

- State that the points that satisfy the above equation are on the surface of the sphere and not on the interior. The graph of the above equation is called a **surface in space.**

- State that the intersection of a surface in space and a plane parallel to one of the coordinate planes is called a **trace** of the surface.

 Example 6. Describe the xz-trace of the sphere whose equation is
 $(x - 4)^2 + (y + 3)^2 + (z - 1)^2 = 25$.
 In the xz-plane, $y = 0$. This yields $(x - 4)^2 + (0 + 3)^2 + (z - 1)^2 = 25$, or $(x - 4)^2 + (z - 1)^2 = 16$. Hence, the xz-trace is a circle of radius 4 with center at (4, 0, 1).