Chapter 12 Limits and an Introduction to Calculus

Course/Section Lesson Number Date

Section 12.1 Introduction to Limits

Section Objectives: Students will know how to estimate limits, use properties of limits, and evaluate limits by direct substitution.

I. The Limit Concept (p. 852) Pace: 1 minute

- State that the notion of a limit of a function is fundamental to all calculus.

II. Definition of Limit (pp. 853−854) Pace: 10 minutes

- State that if $f(x)$ becomes arbitrarily close to a unique number L as x gets closer to c, then the **limit** of $f(x)$ as x approaches c is L. This is denoted by $\lim_{x\to c} f(x) = L$.

Example 1. Use a table to evaluate the following limits.

a) $\lim_{x\to 4} (x-1)$

x	3.9	3.99	4	4.001	4.01	4.1
$f(x)$	2.9	2.99	?	3.001	3.01	3.1

Hence, $\lim_{x\to 4} (x-1) = 3$. Also not that $f(4) = 3$.

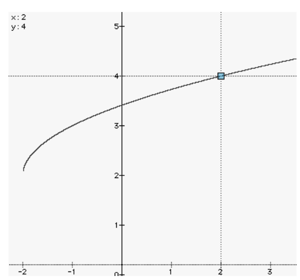
b) $\lim_{x\to 2} \dfrac{x^2-4}{x-2}$

x	1.9	1.99	2	2.001	2.01	2.1
$f(x)$	3.9	3.99	?	4.001	4.01	4.1

Hence, $\lim_{x\to 2} \dfrac{x^2-4}{x-2} = 4$. Also not that f is undefined at 2.

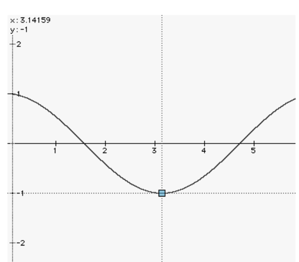
Example 2. Use the graph of the function to evaluate the following limits.

a) $\lim_{x\to 2} \dfrac{x-2}{\sqrt{x+2}-\sqrt{x}}$



Hence, the limit is 4. Note that although it does not appear on our graphing utilities, there is a hole in the graph at $x = 2$.

b) $\lim_{x\to \pi} \cos x$



Hence, the limit is –1. Also note that $\cos \pi = -1$.

III. Limits That Fail to Exist (pp. 855−856) Pace: 10 minutes

Example 3. Do the following limits exist?

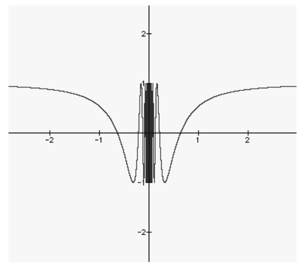
a) $\lim_{x \to 1} \frac{|x-1|}{x-1}$

Since $f(x) = 1$ if $x > 1$ and $f(x) = -1$ if $x < 1$, the limit fails to exist.

b) $\lim_{x \to 0} \frac{1}{x}$

Since $f(x) \to \infty$ if $x > 0$ and $f(x) \to -\infty$ if $x < 0$, the limit fails to exist.

c) $\lim_{x \to 0} \cos \frac{1}{x}$



You can see that as x approaches 0, $f(x)$ oscillates between –1 and 1. Therefore there is no *unique* number that $f(x)$ approaches.

- Direct students' attention to the *Technology* feature at the bottom of text page 856.

IV. Properties of Limits and Direct Substitution (pp. 857−859)
Pace: 15 minutes

- State the following properties of limits.
  1. $\lim_{x \to c} b = b$
  2. $\lim_{x \to c} x = c$
  3. $\lim_{x \to c} x^n = c^n$
  4. $\lim_{x \to c} \sqrt[n]{x} = \sqrt[n]{c}$

- State the following operations with limits.
  1. $\lim_{x \to c} bf(x) = b \lim_{x \to c} f(x)$
  2. $\lim_{x \to c} [f(x) \pm g(x)] = \lim_{x \to c} f(x) \pm \lim_{x \to c} g(x)$
  3. $\lim_{x \to c} [f(x)g(x)] = \left[\lim_{x \to c} f(x)\right]\left[\lim_{x \to c} g(x)\right]$
  4. $\lim_{x \to c} \frac{f(x)}{g(x)} = \frac{\lim_{x \to c} f(x)}{\lim_{x \to c} g(x)}$, if $\lim_{x \to c} g(x) \neq 0$
  5. $\lim_{x \to c} [f(x)]^n = \left[\lim_{x \to c} f(x)\right]^n$

Example 4. Evaluate the following limits.

a) $\lim_{x \to 4} \sqrt{x} = \sqrt{4} = 2$

b) $\lim_{x \to 1} x^2 - 2x + 5 = 1^2 - 2(1) + 5 = 4$

c) $\lim_{x \to 0} \frac{x+1}{x^2+5x-4} = \frac{0+1}{0+0-4} = -\frac{1}{4}$

- State that if *p* is a polynomial function, then $\lim_{x \to c} p(x) = p(c)$.
- State that if $r(x) = p(x)/q(x)$, where *p* and *q* are polynomial functions, then $\lim_{x \to c} r(x) = r(c)$, if $q(c) \neq 0$.

Example 5. Evaluate the following limits.

a) $\lim_{x \to 1} 2x^2 + 6x - 5 = 2(1)^2 + 6(1) - 5 = 3$

b) $\lim_{x \to 2} \frac{x^2 - x - 1}{x^2 + 2} = \frac{2^2 - 2 - 1}{2^2 + 2} = \frac{1}{6}$

- Assign the *Writing About Mathematics* on page 859 of the text.