Chapter 12 Limits and an Introduction to Calculus

Course/Section
Lesson Number
Date

Section 12.2 Techniques for Evaluating Limits

Section Objectives: Students will know how to find limits by using the dividing out and rationalizing techniques, how to evaluate one-sided limits, and how to evaluate limits of difference quotients.

I. Dividing Out Technique (pp. 863–864) Pace: 10 minutes

- State that we now reduce fractions before evaluating the limit when direct substitution yields 0/0.

Example 1. Evaluate the following limits.

a) $\lim_{x\to 3}\frac{x^2-9}{x-3}=\lim_{x\to 3}\frac{(x+3)(x-3)}{x-3}=\lim_{x\to 3}(x+3)=6$

b) $\lim_{x\to 1}\frac{x-1}{x^3-1}=\lim_{x\to 1}\frac{x-1}{(x-1)(x^2+x+1)}=\lim_{x\to 1}\frac{1}{(x^2+x+1)}=\frac{1}{3}$

II. Rationalizing Technique (p. 865) Pace: 5 minutes

- State that we now rationalize fractions involving radicals before evaluating the limit when direct substitution yields 0/0.

Example 2. Evaluate the following limit.

$$\lim_{x\to 2}\frac{\sqrt{x+2}-\sqrt{2}}{x}=\lim_{x\to 2}\frac{\sqrt{x+2}-\sqrt{2}}{x}\cdot\frac{\sqrt{x+2}+\sqrt{2}}{\sqrt{x+2}+\sqrt{2}}$$

$$=\lim_{x\to 2}\frac{x+2-2}{x\left(\sqrt{x+2}+\sqrt{2}\right)}$$

$$=\lim_{x\to 2}\frac{1}{\sqrt{x+2}+\sqrt{2}}$$

$$=\frac{1}{2+\sqrt{2}}$$

III. Using Technology (pp. 866–867) Pace: 15 minutes

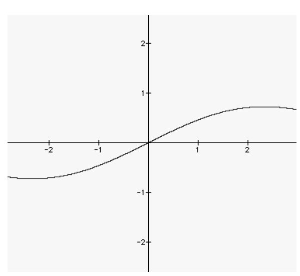
Example 3. Approximate $\lim_{x\to 0}\frac{1-\cos x}{x}$.

Numerical Solution

x	$f(x)$
-0.003	-0.0015
-0.002	-0.0010
-0.001	-0.0005
0	#DIV/0!
0.001	0.0005
0.002	0.0010
0.003	0.0015

Hence the limit is 0.

Graphical Solution



- Draw students' attention to the *Technology* feature at the top of text page 867.

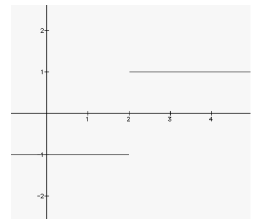
IV. One-Sided Limits (pp. 867–868) Pace: 10 minutes

- Discuss the right-hand and left-hand limit notation, $\lim_{x \to c^+} f(x) = L$ and $\lim_{x \to c^-} f(x) = L$.
- State that $\lim_{x \to c} f(x) = L$ if and only if $\lim_{x \to c^+} f(x) = L$ and $\lim_{x \to c^-} f(x) = L$.

Example 4. Find the limit of $f(x)$ as x approaches 2.

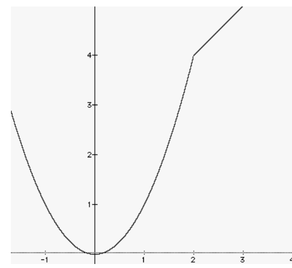
a) $f(x) = |x - 2|/(x - 2)$

From the graph we see that the limit from the right is 1 and the limit from the left is –1. Therefore the limit of $f(x)$ as x approaches 2 fails to exist.



b) $f(x) = \begin{cases} x + 2, x \geq 2 \\ x^2, x < 2 \end{cases}$

The limit of $f(x)$ as x approaches 2 is 4.



V. A Limit from Calculus (p. 869) Pace: 5 minutes

Example 5. For $f(x) = x^2 - x$, find the following limit.

$$\lim_{h \to 0} \frac{f(1+h) - f(1)}{h} = \lim_{h \to 0} \frac{\left[(1+h)^2 - (1+h)\right] - \left[1^2 - 1\right]}{h}$$

$$= \lim_{h \to 0} \frac{1 + 2h + h^2 - h - 1}{h}$$

$$= \lim_{h \to 0} \frac{h + h^2}{h}$$

$$= \lim_{h \to 0} (1 + h)$$

$$= 1$$