Chapter 2 Polynomial and Rational Functions

Course/Section
Lesson Number
Date

Section 2.1 Quadratic Functions and Models

Section Objectives: Students will know how to sketch and analyze graphs of quadratic functions.

I. The Graph of a Quadratic Function (pp. 128−130) Pace: 10 minutes

- Define a **polynomial function** to be a function of the form
  $f(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$
  where each a_i is a real number and n is a nonnegative integer. Note that we have already dealt with two forms of this equation, when $n = 0$ and $n = 1$. In this section we focus on $n = 2$. These are called **quadratic functions** and we simplify the notation to be
  $f(x) = ax^2 + bx + c$, with $a \neq 0$.
  The graph of a quadratic function is a parabola.
- Draw the graph of $y = x^2$ and identify the vertex (at the origin) and the axis. Work the *Exploration* on page 129 of the text.

II. The Standard Form of a Quadratic Function (pp. 131−132)
Pace: 10 minutes

- Apply the material covered in Section 1.7 to produce the following.
  The quadratic function
  $f(x) = a(x - h)^2 + k$, $a \neq 0$
  is in **standard form.** The graph of f is a parabola with vertical axis $x = h$ and with vertex at (h, k). If $a > 0$, the parabola opens upward, and if $a < 0$, the parabola opens downward.

Example 1. Find the vertex of the following parabola.

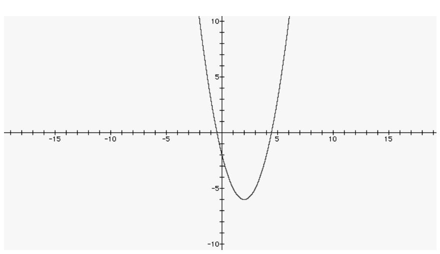
$$\begin{aligned} f(x) &= -2x^2 - 4x + 1 \\ &= -2\left(x^2 + 2x\right) + 1 \\ &= -2\left(x^2 + 2x + 1\right) + 1 + 2 \\ &= -2(x + 1)^2 + 3 \end{aligned}$$

The vertex is at (−1, 3).

Example 2. Graph the following quadratic function.

$$\begin{aligned} f(x) &= x^2 - 4x - 2 \\ &= \left(x^2 - 4x + 4\right) - 4 - 2 \\ &= (x - 2)^2 - 6 \end{aligned}$$

The vertex is at (2, −6), the parabola opens upward, and there is no change in the width.



Example 3. Find the standard form of the equation of the parabola that has vertex at (1, −2) and passes through the point (3, 6).
From the vertex, we have this much of the equation: $f(x) = a(x - 1)^2 - 2$.
To find a we substitute the point (3, 6) and solve for a.

$$6 = a(3 - 1)^2 - 2$$
$$6 = 4a - 2$$
$$8 = 4a$$
$$2 = a$$

The equation is $f(x) = 2(x - 1)^2 - 2$.

III. Applications (p. 133) Pace: 5 minutes

- Graph $y = -(x - 4)^2 + 5$. Ask the class what the minimum value of y is.

Example 4. The daily cost of manufacturing a particular product is given by

$$C(x) = 1200 - 7x + 0.1x^2$$

where x is the number of units produced each day. How many units should be produced to minimize cost?
We need to find h.

$$\begin{aligned} C(x) &= 1200 - 7x + 0.1x^2 \\ &= 0.1(x^2 - 70x) + 1200 \\ &= 0.1(x^2 - 70x + 35^2) + 1200 - 122.5 \\ &= 0.1(x - 35)^2 + 1077.5 \end{aligned}$$

Producing 35 units per day will minimize the cost.