# Chapter 2 Polynomial and Rational Functions

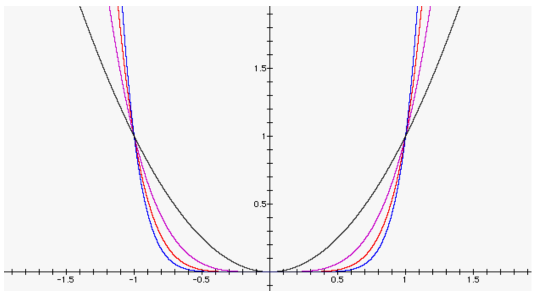
| Course/Section<br>Lesson Number<br>Date |
| --- |

## Section 2.2 Polynomial Functions of Higher Degree

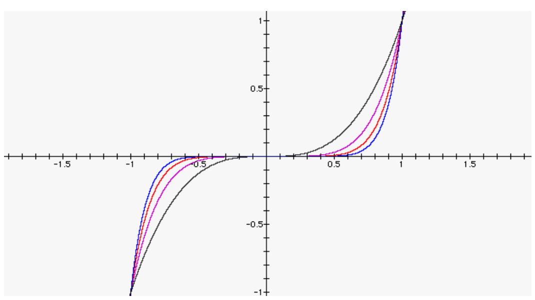
**Section Objectives:** Students will know how to sketch and analyze graphs of polynomial functions.

**I. Graphs of Polynomial Functions** (pp. 139−140) Pace: 10 minutes

- Discuss these characteristics of graphs of polynomial functions.
  1. Polynomial functions are continuous. This means that the graphs of polynomial functions have no breaks, holes, or gaps.
  2. The graphs of polynomial functions have only nice, smooth turns and bends. There are no sharp turns as in the graph of $y = |x|$.
- We will first look at the simplest polynomials, $f(x) = x^n$. These are called **power functions.** We can break these into two cases, $n$ is even and $n$ is odd.



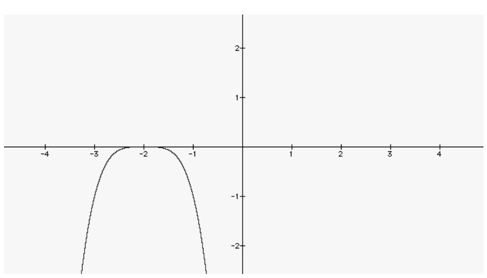
Here $n$ is even. Note how the graph flattens at the origin as $n$ increases.



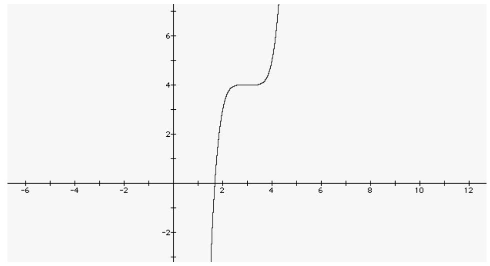
Here $n$ is odd. Note how the graph flattens at the origin as $n$ increases.

**Example 1.** Sketch the graph of the following.

**a)** $f(x) = -(x + 2)^4$



**b)** $f(x) = (x - 3)^5 + 4$



**II. The Leading Coefficient Test** (pp. 141−142) Pace: 10 minutes

- Work the *Exploration* on page 141 of the text. Then summarize with the following chart.

| $f(x) = a_n x^n + \cdots$ | $a_n > 0$ | $a_n < 0$ |
|---|---|---|
| $n$ even | ↖ ↗ | ↙ ↘ |
| $n$ odd | ↙ ↗ | ↖ ↘ |

**Example 2.** Describe the right-hand and left-hand behavior of the graph of each function.

**a)** $f(x) = -x^4 + 7x^3 - 14x - 9$
Down to both sides

**b)** $g(x) = 5x^5 + 2x^3 - 14x^2 + 6$
Down to the left and up to the right

**III. Zeros of Polynomial Functions** (pp. 142−145) Pace: 15 minutes

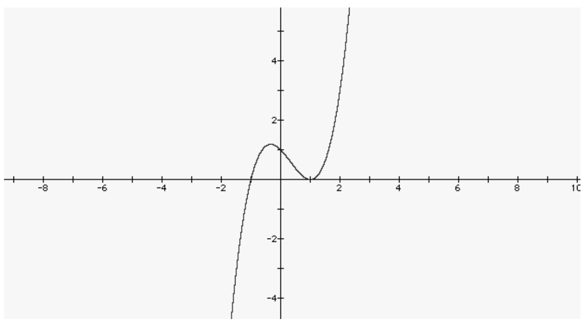
- State the following as being equivalent statements, where $f$ is a polynomial function and $a$ is a real number.
  1. $x = a$ is a zero of $f$.
  2. $x = a$ is a solution of the equation $f(x) = 0$.
  3. $(x - a)$ is a factor of $f(x)$.
  4. $(a, 0)$ is an $x$-intercept of the graph of $f$.

**Example 3.** Find the *x*-intercepts of the graph of $f(x) = x^3 - x^2 - x + 1$.

$$f(x) = x^3 - x^2 - x + 1$$
$$0 = x^2(x-1) - 1(x-1)$$
$$0 = (x^2 - 1)(x-1)$$
$$0 = (x-1)^2(x+1)$$
$$(x-1)^2 = 0 \Rightarrow x = 1$$
$$x + 1 = 0 \Rightarrow x = -1$$

The *x*-intercepts are (−1, 0) and (1, 0).

- Note that in the above example, 1 is a repeated zero. In general, a factor $(x - a)^k$, $k > 1$, yields a **repeated zero** $x = a$ of **multiplicity** $k$. If $k$ is odd, the graph *crosses* the *x*-axis at $x = a$. If $k$ is even, the graph only *touches* the *x*-axis at $x = a$. See the graph below.



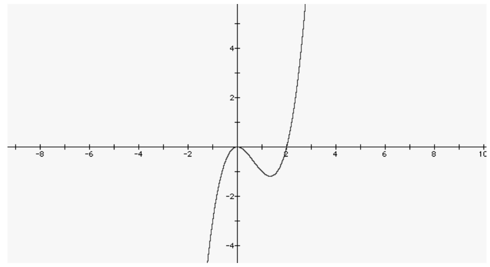

- To graph a polynomial function, you can use the fact that the function can change signs only at its zeros. Between two consecutive zeros, the polynomial must be either *entirely positive* or *entirely negative*. Tell students that if the real zeros are put in order, they divide the number line into **test intervals** on which the function has no sign changes. By picking a representative x-value in each test interval, students can determine whether that portion of the graph lies above the x-axis (positive value of *f*) or below the x-axis (negative value of *f*).

**Example 4.** Sketch the graph of $f(x) = x^3 - 2x^2$.

1. Since $f(x) = x^2(x - 2)$, the *x*-intercepts are (0, 0) and (2, 0). Also, 0 has multiplicity 2; therefore, the graph will just touch at (0, 0).
2. The graph will go up to the right and down to the left.
3. Use the zeros of the polynomial to find test intervals. Additional points on the graph are (−1, −3), (1, −1), and (3, 9).

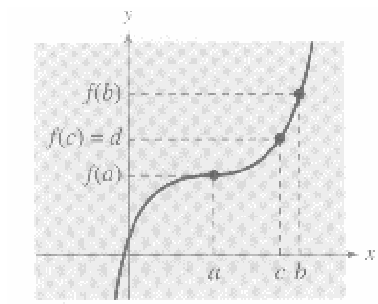
| *Test Interval* | $(-\infty, 0)$ | (0, 2) | $(2, \infty)$ |
|---|---|---|---|
| *x-value* | −1 | 1 | 3 |
| *Result* | $f(-1) = -3$<br>Negative | $f(1) = -1$<br>Negative | $f(3) = 9$<br>Positive |

4. Sketch the graph.



**IV. The Intermediate Value Theorem** (pp. 146−147) Pace: 10 minutes

- Draw and label a picture similar to the one on page 146 of the text, which has been included here.




- Show how the Intermediate Value Theorem, which follows here, applies.

  Let $a$ and $b$ be real numbers such that $a < b$. If $f$ is a polynomial function such that $f(a) \neq f(b)$, then, in the interval $[a, b]$, $f$ takes on every value between $f(a)$ and $f(b)$.

- We can use this theorem to approximate zeros of polynomial functions if $f(a)$ and $f(b)$ have different signs.

**Example 5.** Use the Intermediate Value Theorem to approximate the real zero of $f(x) = 4x^3 - 7x^2 - 21x + 18$ on $[0, 1]$.

Note that $f(0) = 18$ and $f(1) = -6$. Therefore, by the Intermediate Value Theorem, there must be a zero between 0 and 1.
Furthermore, note that $f(0.7) = 1.242$ and $f(0.8) = -1.232$. Therefore there must be a zero between 0.7 and 0.8.
This process can be repeated until the desired accuracy is obtained.