Chapter 2 Polynomial and Rational Functions

Course/Section
Lesson Number
Date

Section 2.3 Polynomial and Synthetic Division

Section Objectives: Students will know how to use long division and synthetic division to divide polynomials by other polynomials.

I. Long Division of Polynomials (pp. 153−155) Pace: 10 minutes

- Review long division of integers. Use 7213 ÷ 61 as an illustration. Now divide polynomials the same way.

Tip: Many students have trouble with the subtraction; they want to add. They will need to be reminded to subtract.

Example 1. Divide $2x^3 - 5x^2 + x - 8$ by $x - 3$.

$$\begin{array}{r} 2x^2 + x + 4 \\ x - 3 \overline{)\,2x^3 - 5x^2 + x - 8} \\ \underline{2x^3 - 6x^2} \qquad\qquad \\ x^2 + x \qquad \\ \underline{x^2 - 3x} \qquad \\ 4x - 8 \\ \underline{4x - 12} \\ 4 \end{array}$$

The result is $2x^2 + x + 4 + \dfrac{4}{x - 3}$.

- State the **Division Algorithm**.
  For all polynomials $f(x)$ and $d(x)$ such that the degree of d is less than or equal to the degree of f and $d(x) \neq 0$, there exist unique polynomials $q(x)$ and $r(x)$ such that
  $f(x) = d(x)q(x) + r(x)$
  where $r(x) = 0$ or the degree of r is less than the degree of d.

  $$\frac{f(x)}{d(x)} = q(x) + \frac{r(x)}{d(x)}$$

  This is why we write the remainder the way we do.

Tip: Tell the class to never use the lowercase "r" for the remainder; some students will still want to use it.

Example 2. Divide $3x^3 - x^2 + 2x - 3$ by $x - 2$.

$$\begin{array}{r} 3x^2 + 5x + 12 \\ x - 2 \overline{)\,3x^3 - x^2 + 2x - 3} \\ \underline{3x^3 - 6x^2} \qquad\qquad \\ 5x^2 + 2x \qquad \\ \underline{5x^2 - 10x} \qquad \\ 12x - 3 \\ \underline{12x - 24} \\ 21 \end{array}$$

The result is $3x^2 + 5x + 12 + \dfrac{21}{x - 2}$.

II. Synthetic Division (p. 156) Pace: 10 minutes

- Use the preceding example to develop synthetic division as follows.
  1. State that the following procedure applies only when the divisor is of the form $x - c$, and when every descending power of x has a place in the dividend.

  2. Eliminate all the variables, since we are keeping everything in nice columns.

$$\begin{array}{r|rrrr}
\multicolumn{1}{r}{} & & 3 & 5 & 12 \\ \cline{2-5}
1\ -2 & 3 & -1 & 2 & -3 \\
 & \underline{3} & \underline{-6} & & \\
 & & 5 & 2 & \\
 & & \underline{5} & \underline{-10} & \\
 & & & 12 & -3 \\
 & & & \underline{12} & \underline{-24} \\
 & & & & 21
\end{array}$$

3. Now eliminate the 1 in the divisor because, to do synthetic division, that coefficient must always be 1. Also, eliminate the numbers that are, by design, the same as the numbers directly above them.

$$\begin{array}{r|rrrr}
\multicolumn{1}{r}{} & & 3 & 5 & 12 \\ \cline{2-5}
-2 & 3 & -1 & 2 & -3 \\
 & & \underline{-6} & & \\
 & & 5 & 2 & \\
 & & & \underline{-10} & \\
 & & & 12 & -3 \\
 & & & & \underline{-24} \\
 & & & & 21
\end{array}$$

4. Next eliminate the numbers that we "bring down."

$$\begin{array}{r|rrrr}
\multicolumn{1}{r}{} & & 3 & 5 & 12 \\ \cline{2-5}
-2 & 3 & -1 & 2 & -3 \\
 & & \underline{-6} & & \\
 & & 5 & & \\
 & & & \underline{-10} & \\
 & & & 12 & \\
 & & & & \underline{-24} \\
 & & & & 21
\end{array}$$

5. Now each column has a number, a line, and another number. Let us merge all of the lines.

$$
\begin{array}{r|rrrr}
 & & 3 & 5 & 12 \\
-2) & 3 & -1 & 2 & -3 \\
 & & -6 & -10 & -24 \\
\hline
 & & 5 & 12 & 21
\end{array}
$$

6. Note that if we place a 3 at the start of the last row and isolate the 21, the top and bottom rows will look the same. So, eliminate the top row.

$$
\begin{array}{r|rrrr}
-2) & 3 & -1 & 2 & -3 \\
 & & -6 & -10 & -24 \\
\hline
 & 3 & 5 & 12 & |21
\end{array}
$$

7. The last thing to do is to eliminate the subtraction, since we prefer to add. We need to change the sign of everything in the second row. We can achieve this by changing the sign of the divisor, since everything in the second row is a multiple of the divisor.

$$
\begin{array}{r|rrrr}
2) & 3 & -1 & 2 & -3 \\
 & & 6 & 10 & 24 \\
\hline
 & 3 & 5 & 12 & |21
\end{array}
$$

Example 3. Use synthetic division to divide $4x^4 - 2x^2 - x + 1$ by $x + 2$.

$$
\begin{array}{r|rrrrr}
-2 & 4 & 0 & -2 & -1 & 1 \\
 & & -8 & 16 & -28 & 58 \\
\hline
 & 4 & -8 & 14 & -29 & 59
\end{array}
$$

The result is $4x^3 - 8x^2 + 14x - 29 + \dfrac{59}{x+2}$.

III. The Remainder and Factor Theorems (pp. 157−158) Pace: 15 minutes

- State the **Remainder Theorem.**
  If a polynomial $f(x)$ is divided by $x - k$, the remainder is $r = f(k)$.

Tip: If you have time you should prove this theorem. A proof is given on page 213 of the text.

Example 4. Use the Remainder Theorem to find $f(1)$.

$f(x) = x^3 - 2x^2 - 4x + 1$

$$
\begin{array}{r|rrrr}
1 & 1 & -2 & -4 & 1 \\
 & & 1 & -1 & -5 \\
\hline
 & 1 & -1 & -5 & -4
\end{array}
$$

$f(1) = -4$

- State the **Factor Theorem.**
  A polynomial $f(x)$ has a factor $x - k$, if and only if $f(k) = 0$.

  Tip: A proof of the Factor Theorem is given on page 213 of the text.

  Example 5. Show that $x - 1$ is a factor of $f(x) = x^4 - 1$.

  $$f(1) = 1^4 - 1 = 0$$

  Because $f(1) = 0$, by the Factor Theorem, $x - 1$ is a factor of $f(x)$.