Chapter 2 Polynomial and Rational Functions

Course/Section Lesson Number Date

Section 2.4 Complex Numbers

Section Objectives: Students will know how to work with complex numbers and solve quadratic equations with complex solutions.

I. The Imaginary Unit *i* (p. 162) Pace: 5 minutes

- We need a new set of numbers because simple equations such as $x^2 + 1 = 0$ do not have real solutions. We need a number whose square is –1. So we define $i^2 = -1$. i is called the **imaginary unit**. By adding real numbers to multiples of the imaginary unit, we get the set of **complex numbers**, defined as $\{a + bi \mid a \text{ is real, } b \text{ is real, and } i^2 = -1\}$. $a + bi$ is the **standard form** of a complex number. a is called the **real part** and bi is called the **imaginary part**.
- Two complex numbers $a + bi$ and $c + di$ are equal to each other if and only if $a = c$ and $b = d$.

II. Operations with Complex Numbers (pp.163–164) Pace: 10 minutes

- To add two complex numbers, we add the two real parts and then add the two imaginary parts. That is,
$$(a + bi) + (c + di) = (a + c) + (b + d)i.$$
- Note that the **additive identity** of the complex numbers is zero. Furthermore, the **additive inverse** of $a + bi$ is $-(a + bi) = -a - bi$.

Example 1. Add or subtract the following complex numbers.

a) $(6-3i)+(1+2i)=(6+1)+(-3+2)i=7-i$

b) $(5-i)-(2-4i)=5-i-2+4i=3+3i$

- Many of the properties of real numbers are valid for complex numbers as well, including both of the Associative and Commutative Properties and both Distributive Properties.
- Using these properties, much as we did with polynomials, is the best method for multiplying two complex numbers.
- Now is a good time to examine the Exploration on page 164 of the text.

Example 2. Multiply the following complex numbers.

a) $2(17-5i)=2(17)-2(5i)=34-10i$

b)
$$\begin{aligned}(3-i)(5+4i) &= 15+12i-5i-4i^2\\ &= 15+12i-5i+4\\ &= 19+7i\end{aligned}$$

c) $(1+7i)^2=1^2+2(1)(7i)+(7i)^2=1+14i-49=-48+14i$

d) $(4+5i)(4-5i)=4^2-(5i)^2=16+25=41$

Tip: Note that in the last example, we took the product of two complex numbers and got a real number. This leads to our next topic.

III. Complex Conjugates (p. 165) Pace: 5 minutes

- Two complex numbers of the forms $a + bi$ and $a - bi$ are called **complex conjugates**. Note that their product is a real number:
$$(a + bi)(a - bi) = a^2 - (bi)^2 = a^2 + b^2$$
We will now use this fact to write a quotient of complex numbers in standard form.

Example 3. Write the following quotients in standard form.

a) $\dfrac{2}{1-i}=\dfrac{2}{1-i}\cdot\dfrac{1+i}{1+i}=\dfrac{2(1+i)}{1^2+1^2}=\dfrac{2(1+i)}{2}=1+i$

b)

$$\begin{aligned}\frac{2-i}{4+3i}&=\frac{2-i}{4+3i}\cdot\frac{4-3i}{4-3i}\\&=\frac{8-6i-4i+3i^2}{4^2+3^2}\\&=\frac{8-3-6i-4i}{16+9}\\&=\frac{5-10i}{25}\\&=\frac{1}{5}-\frac{2}{5}i\end{aligned}$$

IV. Complex Solutions of Quadratic Equations (p. 166) Pace: 10 minutes

- We now use the principal square root of a negative number to solve quadratic equations with complex solutions. The **principal square root** of a negative number is defined by $\sqrt{-a}=\sqrt{a}i$, where $a > 0$.

Example 4. Rewrite the following in standard form.

a) $\sqrt{-8}\sqrt{-6}=\left(2\sqrt{2}i\right)\left(\sqrt{6}i\right)=2\sqrt{12}i^2=-4\sqrt{3}$

b)

$$\begin{aligned}\left(1-\sqrt{-14}\right)^2&=\left(1-\sqrt{14}i\right)^2\\&=1^2-2(1)\sqrt{14}i+\left(\sqrt{14}i\right)^2\\&=1-2\sqrt{14}i-14\\&=-13-2\sqrt{14}i\end{aligned}$$

Example 5. Solve.

a)

$$\begin{aligned}x^2+9&=0\\x^2&=-9\\x&=\pm\sqrt{-9}=\pm 3i\end{aligned}$$

b)

$$\begin{aligned}x^2+6x+15&=0\\x^2+6x&=-15\\x^2+6x+3^2&=-15+3^2\\(x+3)^2&=-6\\x+3&=\pm\sqrt{6}i\\x&=-3\pm\sqrt{6}i\end{aligned}$$