Chapter 2 Polynomial and Rational Functions

Course/Section
Lesson Number
Date

Section 2.6 Rational Functions

Section Objectives: Students will know how to determine the domains of, find the asymptotes of, and sketch the graphs of rational functions.

I. Introduction (p. 184) Pace: 5 minutes

- State the following definition.
  A **rational function** is a function of the form $f(x) = N(x)/D(x)$, where N and D are both polynomials. The domain of f is all x such that $D(x) \neq 0$.

Example 1. Find the domain of $f(x)=\dfrac{2x+1}{x^2-4}$.

$$x^2-4=0$$
$$(x+2)(x-2)=0$$
$$x+2=0 \Rightarrow x=-2$$
$$x-2=0 \Rightarrow x=2$$

The domain is $(-\infty, -2) \cup (-2, 2) \cup (2, \infty)$.

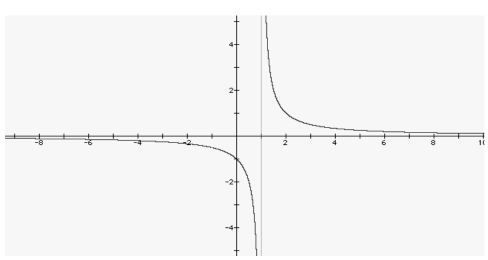
II. Horizontal and Vertical Asymptotes (pp. 185−186) Pace: 15 minutes

- Use the graph of $y=\dfrac{2x-5}{x-2}$ to discuss vertical and horizontal asymptotes.
- State the following definitions of asymptotes.
  1. The line $x = a$ is a **vertical asymptote** of the graph of f if $f(x) \to \pm\infty$ as $x \to a$, either from the right or from the left.
  2. The line $y = b$ is a **horizontal asymptote** of the graph of f if $f(x) \to b$ as $x \to \pm\infty$.
- State the following **Rules for Asymptotes of Rational Functions.**
  Let f be a rational function given by
  $$f(x)=\frac{N(x)}{D(x)}=\frac{a_nx^n+a_{n-1}x^{n-1}+\cdots+a_1x+a_0}{b_nx^n+b_{n-1}x^{n-1}+\cdots+b_1x+b_0}.$$
  1. The graph of f has a *vertical* asymptote at $x = a$ if $D(a) = 0$ and $N(a) \neq 0$.
  2. The graph of f has one *horizontal* asymptote or no horizontal asymptote, depending on the degree of N and D.
     a. If $n < m$, then $y = 0$ is the horizontal asymptote of the graph of f.
     b. If $n = m$, then $y = a_n/b_m$ is the horizontal asymptote of the graph of f.
     c. If $n > m$, then there is no horizontal asymptote of the graph of f.

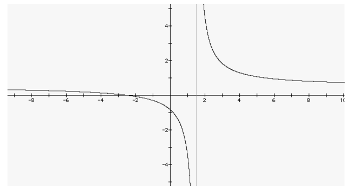
Example 2. Find any horizontal and vertical asymptotes of the following.

a) $f(x)=\dfrac{x+1}{x^2-1}$

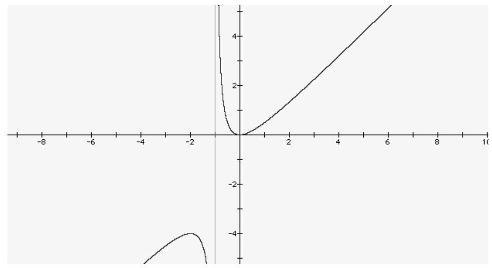
The horizontal asymptote is $y = 0$. The only vertical asymptote is $x = 1$. There will be a hole in the graph at $x = -1$.



b) $g(x) = \dfrac{2x+5}{4x-6}$. The horizontal asymptote is at $y = 1/2$, and the vertical asymptote is at $x = 3/2$.



c) $h(x) = \dfrac{x^2}{x+1}$. No horizontal asymptote and a vertical asymptote at $x = -1$.



III. Analyzing Graphs of Rational Functions (pp. 187–189) Pace: 15 minutes

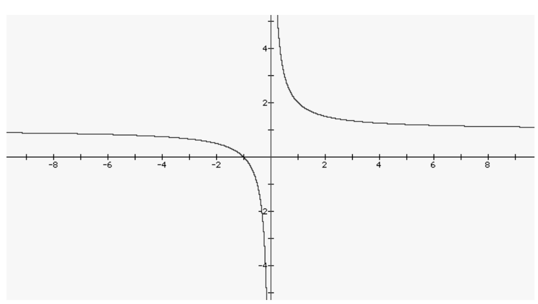
Draw attention to the **Guidelines for Analyzing Graphs of Rational Functions** and the *Technology* feature on page 187 of the text.

Example 3. Sketch the graph of each of the following functions.

a) $f(x) = \dfrac{x+1}{x}$

y-Intercept:	None
x-Intercept:	$(-1, 0)$
Vertical asymptote:	$x = 0$
Horizontal asymptote:	$y = 1$
Additional points:	$(-2, 0.5)$, $(-0.5, -1)$, $(1, 2)$

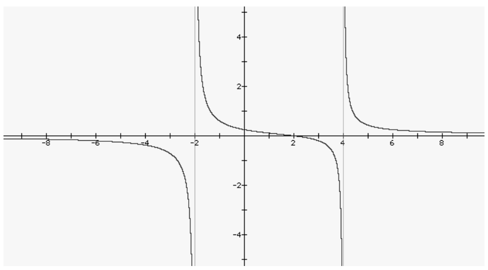
Test Interval	$(-\infty, -1)$	$(-1, 0)$	$(0, \infty)$
x-value	-2	-0.5	1
Result	$f(-2) = 0.5$ Positive	$f(-0.5) = -1$ Negative	$f(1) = 2$ Positive



b) $g(x) = \dfrac{x-2}{x^2 - 2x - 8}$

y-Intercept:	(0, 0.25)
x-Intercept:	(2, 0)
Vertical asymptotes:	$x = -2$ and $x = 4$
Horizontal asymptote:	$y = 0$
Additional points:	(−3, −0.714), (0, 1/4), (3, −0.2), (6, 1/4)

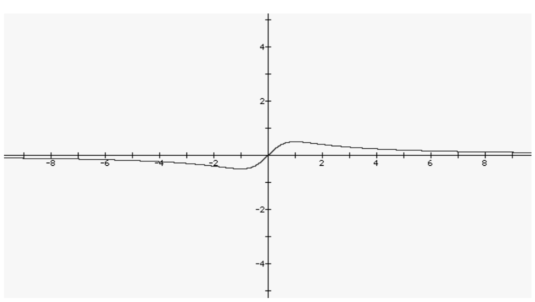
Test Interval	(−∞, −2)	(−2, 2)	(2, 4)	(4, ∞)
x-value	−3	0	3	6
Result	$f(-3) = -0.714$ Negative	$f(0) = 1/4$ Positive	$f(3) = -0.2$ Negative	$f(6) = 1/4$ Positive



c) $h(x) = \dfrac{x}{x^2 + 1}$

y-Intercept:	(0, 0)
x-Intercept:	(0, 0)
Vertical asymptote:	none
Horizontal asymptote:	$y = 0$
Additional points:	(−2, −0.4), (−1, −1/2), (1, 1/2)

Test Interval	(−∞, 0)	(0, ∞)
x-value	−1	1
Result	$f(-1) = -1/2$ Negative	$f(1) = 1/2$ Positive



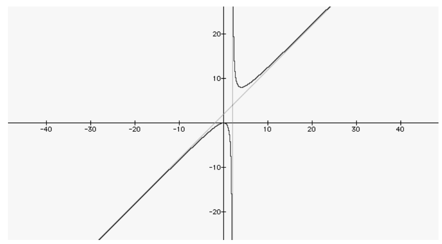
IV. Slant Asymptotes (p. 190) Pace: 10 minutes

- Add one more rule to the **Rules for Asymptotes of a Rational Function**: If $n = m + 1$, then the graph of f has a slant asymptote at $y = q(x)$, where $q(x)$ is the quotient obtained from the division algorithm.

Example 4. Sketch the graph of $y = \frac{x^2}{x-2} = x + 2 + \frac{4}{x-2}$.

y-Intercept: (0, 0)
x-Intercept: (0, 0)
Vertical asymptote: $x = 2$
Slant asymptote: $y = x + 2$
Additional points: (−1, −1/3), (1, −1), (3, 9)

Test Interval	$(-\infty, 0)$	(0, 2)	$(2, \infty)$
x-value	−1	1	3
Result	$f(-1) = -1/3$ Negative	$f(1) = -1$ Negative	$f(3) = 9$ Positive

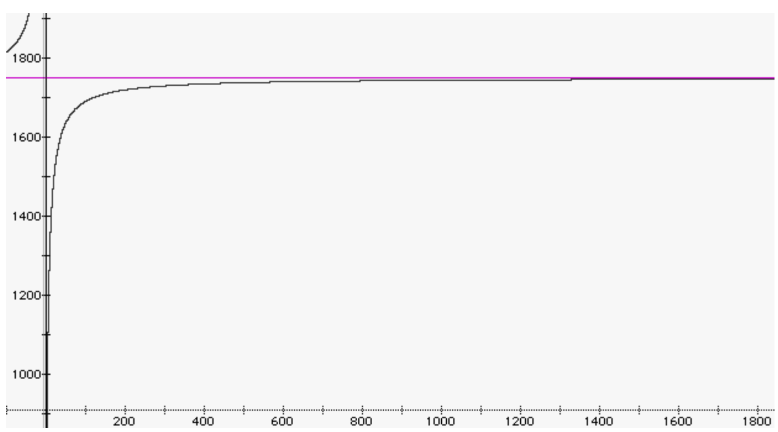


V. Applications (pp. 191−192) Pace: 5 minutes

Example 5. A game commission has determined that if 500 deer are introduced into a preserve, the population at any time t (in months) is given by

$$N = \frac{500 + 350t}{1 + 0.2t}.$$

What is the carrying capacity of the preserve?



The carrying capacity will be equal to the y-value of the horizontal asymptote, $y = 1750$.