Chapter 2 Polynomial and Rational Functions

Course/Section
Lesson Number
Date

Section 2.7 Nonlinear Inequalities

Section Objectives: Students will know how to solve polynomial inequalities and rational inequalities.

I. Polynomial Inequalities (pp. 197−200) Pace: 15 minutes

- Graph a polynomial such as $y = x^2 + x - 6$ using a graphing utility. Explain that there are no sign changes between consecutive zeros of the polynomial. Hence, to solve a polynomial inequality, we need to find the zeros of the polynomial, called the **critical numbers** of the polynomial, use them to create **test intervals,** and test a number from each test interval in the original inequality.

Example 1. Solve the following inequalities.

a) $x^2 + x - 6 > 0$

$(x+3)(x-2) > 0$ The critical numbers are –3 and 2.

Interval	$(-\infty, -3)$	$(-3, 2)$	$(2, \infty)$
x-value	-4	0	3
Result	$6 > 0$	$-6 > 0$ No	$6 > 0$

The solution set is $(-\infty, -3) \cup (2, \infty)$.

b) $x^3 - 4x^2 - x \le -4$

$x^3 - 4x^2 - x + 4 \le 0$

$(x+1)(x-1)(x-4) \le 0$

The critical numbers are 1, −1, and 4.

Interval	$(-\infty, -1)$	$(-1, 1)$	$(1, 4)$	$(4, \infty)$
x-value	-2	0	2	5
Result	$-22 \le -4$	$0 \le -4$ No	$-10 \le -4$	$20 \le -4$ No

The solution set is $(-\infty, -1] \cup [1, 4]$.

Tip: You can check these solutions by graphing the polynomial with a graphing utility. While using the graphing utility, you might want to find the unusual solution sets obtained by solving the following inequalities.

a) $x^2 + 4x + 1 > 0$
b) $x^2 - 4x + 1 \ge 0$
c) $x^2 + 2x + 1 \le 0$
d) $x^2 + 2x + 1 > 0$

II. Rational Inequalities (p. 201) Pace: 10 minutes

- Explain that the concepts of critical numbers and test intervals can be extended to rational inequalities with one exception: a rational expression can also change signs at its undefined values. Therefore, there are two types of critical numbers for rational inequalities.

Example 2. Solve.

$$\frac{2}{x-1} \geq -1$$

$$\frac{2}{x-1} + \frac{x-1}{x-1} \geq 0$$

$$\frac{x+1}{x-1} \geq 0$$

Interval	$(-\infty, -1)$	$(-1, 1)$	$(1, \infty)$
x-value	−2	0	2
Result	$1/3 \geq 0$	$-1 \geq 0$ No	$3 \geq 0$

The solution set is $(-\infty, -1] \cup (1, \infty)$.

III. Applications (pp. 202−203) Pace: 5 minutes

Example 3. The path of a projectile, fired upward from ground level with an initial velocity of 352 feet per second, can be modeled by the equation

$$h = -16t^2 + 352t$$

where h is the height of the projectile in feet and t is time in seconds. During which interval of time is the projectile higher than 1600 feet?

Equation: $-16t^2 + 352t > 1600$

$$t^2 - 22t < -100$$

$$t^2 - 22t + 100 < 0$$

Use the Quadratic Formula to solve for t with $a = 1$, $b = -22$, $c = 100$.
By the Quadratic Formula, $t = 11 \pm 4.58 = 15.58$ or 6.42.
Therefore, the projectile is higher than 1600 feet between 6.42 and 15.48 seconds.