Chapter 3 Exponential and Logarithmic Functions

Course/Section Lesson Number Date

Section 3.1 Exponential Functions and Their Graphs

Section Objectives: Students will know how to recognize, graph, and evaluate exponential functions.

I. Exponential Functions (p. 218) Pace: 5 minutes

- State that we now take a look at our first transcendental function. The **exponential function *f* with base *a*** is denoted by $f(x) = a^x$, where $a > 0$, $a \neq 1$, and x is any real number.
- State that we already know how to work with exponents from the following sets: natural numbers, integers, and rational numbers. To have a complete definition of an irrational exponent we need calculus. We will now look at the graphs of exponential functions and see that nothing strange happens when x is irrational.

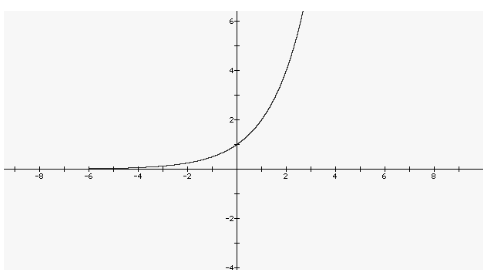
II. Graphs of Exponential Functions (pp. 219−221) Pace: 10 minutes

- State the following facts about the graphs of exponential functions. These all come from considering the equation $f(x) = a^x$.
  1. The domain is $(-\infty, \infty)$.
  2. The range is $(0, \infty)$.
  3. The y-intercept is $(0, 1)$.
  4. $y = 0$ is a horizontal asymptote.
  5. f is increasing if $a > 1$.
  6. f is decreasing if $0 < a < 1$.

Example 1. Graph the following exponential functions.

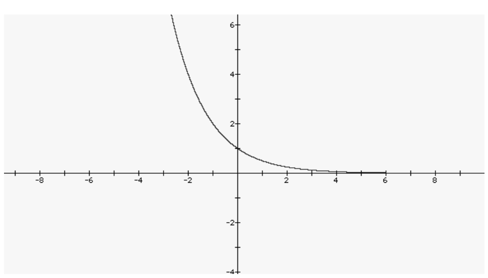
a) $f(x) = 2^x$

Additional points: $(-2, 1/4)$, $(-1, 1/2)$, $(1, 2)$, $(2, 4)$

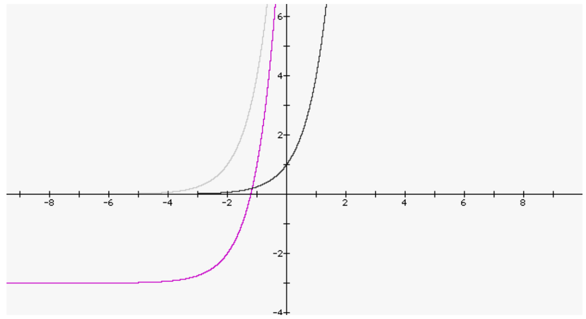


b) $f(x) = 2^{-x} = (1/2)^x$

Additional points: $(2, 1/4)$, $(1, 1/2)$, $(-1, 2)$, $(-2, 4)$



Example 2. Graph each of the following on the same coordinate axes.
a) $g(x) = 4^x$
b) $f(x) = 4^{x+2}$
c) $h(x) = 4^{x+2} - 3$



III. The Natural Base *e* (p. 222) Pace: 5 minutes

- We are going to look at a new number. This number is irrational, so, as is the case with π, we need a symbol for it. We use e, where $e \approx 2.7182818\ldots$ The exponential function with base e is called the **natural exponential function.**

Example 3. Use a calculator to complete the following table.

x	1	10	100	1000	10,000	100,000
$(1+1/x)^x$						

Note that $(1 + 1/x)^x \to e$ as $x \to \infty$.

IV. Applications (pp. 223–225) Pace: 15 minutes

- Develop compound interest as follows. Let A_n be the amount in the account after n pay periods, let P be the principal, and let x be the periodic interest rate.

$$A_1 = P + Px = P(1+x)$$
$$A_2 = P(1+x) + P(1+x)x = P(1+x)(1+x) = P(1+x)^2$$
$$A_3 = P(1+x)^2 + P(1+x)^2 x = P(1+x)^2(1+x) = P(1+x)^3$$
$$\vdots$$
$$A_n = P(1+x)^n$$

- State the compound interest formula

$$A = P(1 + r/n)^{nt}$$

A is the amount in the account after t years.
P is the principal.
r is the annual interest rate.
n is the number of pay periods per year.

Example 4. An investment of $5,000 is made into an account that pays 6% annually for 10 years. Find the amount in the account if the interest is compounded:
a) annually ($n = 1$).

b) quarterly ($n = 4$).

c) monthly ($n = 12$).

d) daily ($n = 365$).

1	4	12	365
\$ 8,954.24	\$ 9,070.09	\$ 9,096.98	\$ 9,110.14

- Note that as n increases, so does A, but the rate of increase slows. Ask the class what would happen if $n \to \infty$. Derive the following **continuously compounded interest formula**.

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

replace n/r by m

$$A = P\left(1 + \frac{1}{m}\right)^{mrt}$$

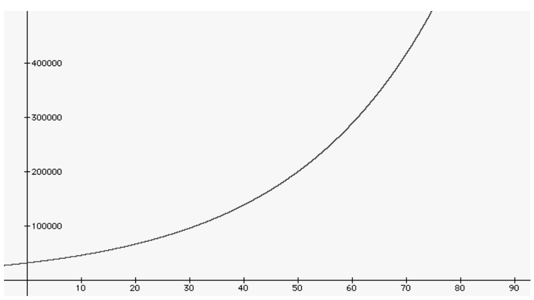
$$A = P\left[\left(1 + \frac{1}{m}\right)^{m}\right]^{rt}$$

Note that as $n \to \infty$, $m \to \infty$; therefore, $A = Pe^{rt}$ as $n \to \infty$.

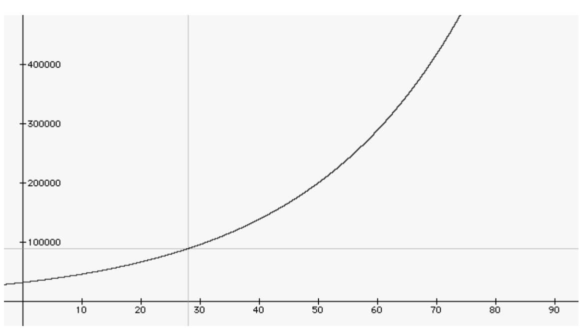
- State the compound interest formula
  $A = Pe^{rt}$
  A is the amount in the account after t years.
  P is the principal.
  r is the annual interest rate.

Example 5. Continue Example 4 above with the interest compounded continuously.
A = \$9,110.59

Example 6. The population of a city increases according to the model $P(t) = 32{,}000e^{0.0367t}$, where $t = 0$ corresponds to 1980. Graph this model and then use it to predict the population in 2008.



$P(28) = 32{,}000e^{0.0367(28)} \approx 89{,}419$



- Assign the *Writing About Mathematics* on page 225 of the text.