Chapter 3 Exponential and Logarithmic Functions

Course/Section
Lesson Number
Date

Section 3.2 Logarithmic Functions and Their Graphs

Section Objectives: Students will know how to recognize, graph, and evaluate logarithmic functions.

I. Logarithmic Functions (pp. 229−231) Pace: 10 minutes

- State that since the exponential function is one-to-one, its inverse is a function. The function given by $f(x) = \log_a x$, where $x > 0$, $a > 0$, and $a \neq 1$, is called the **logarithmic function with base *a***. Furthermore, the logarithmic function with base a is the inverse of the exponential function with base a; thus $y = \log_a x$ if and only if $x = a^y$.

  Example 1. Evaluate each of the following.
  a) $\log_2 8 = 3$
  b) $\log_2 0.25 = -2$
  c) $\log_3 81 = 4$

- State that because we are working in a base number system, we call the logarithmic function with base 10 the **common logarithmic function**. This is the function that corresponds to the LOG button on our calculators. The common logarithmic function is the one function for which we need not write the base.
- State that the following properties follow directly from the definition of the logarithmic function.
  1. $\log_a 1 = 0$
  2. $\log_a a = 1$
  3. $\log_a a^x = x$ and $a^{\log_a x} = x$
  4. If $\log_a x = \log_a y$, then $x = y$.

  Example 2. Solve the following equations.
  a)
  $\log_5 x = \log_5 8$
  $x = 8$

  b)
  $\log_5 1 = x$
  $0 = x$

II. Graphs of Logarithmic Functions (pp. 231−232) Pace: 5 minutes

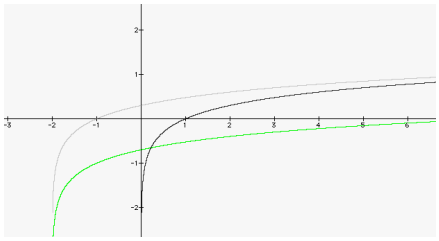
- State that the following properties of the **graph of the logarithmic function** come directly from the properties of the graph of the exponential function and the inverse relationship.
  1. The domain is $(0, \infty)$.
  2. The range is $(-\infty, \infty)$.
  3. The x-intercept is $(1, 0)$.
  4. The y-axis is a vertical asymptote.
  5. The function is increasing ($a > 0$).

Example 3. Sketch the graph of the following on the same coordinate axes.

a) $y = \log_{10} x$

b) $y = \log_{10}(x + 2)$

c) $y = \log_{10}(x + 2) - 1$



III. The Natural Logarithmic Function (pp. 233−234) Pace: 10 minutes

- State that the logarithmic function with base e is called the **natural logarithmic function** and is denoted by $f(x) = \ln x$.

Tip: Students will have trouble remembering that $\ln x = \log_e x$; hence extra emphasis may be required.

Example 4. Evaluate the following.

a) $\ln e^5 = 5$

b) $e^{\ln 3} = 3$

c) $\ln \dfrac{1}{e^2} = -2$

Example 5. Find the domains of the following functions.

a) $f(x) = \ln (x + 3) \Rightarrow x + 3 > 0 \Rightarrow x > -3$

The domain is $(-3, \infty)$.

b) $f(x) = \ln |x| \Rightarrow |x| > 0$

The domain is $(-\infty, \infty)$.

IV. Application (p. 235) Pace: 10 minutes

Example 6. The model

$$t = 12.542 \ln\left(\frac{x}{x - 1000}\right), \; x > 1000$$

approximates the length of a home mortgage of \$150,00 at 8% interest in terms of the monthly payment. In the model, t is the length of the mortgage in years and x is the monthly payment in dollars. Find the length of the home mortgage of \$150,000 at 8% if the monthly payment is \$1300, and find the total interest charged over the life of the loan.

$t = 12.542 \ln\left(\dfrac{1300}{1300 - 1000}\right) \approx 18.4$ years.

Total interest = $18.4 \cdot 1300 \cdot 12 - 150{,}000 = \$137{,}040$