

Chapter 3 Exponential and Logarithmic Functions

Course/Section Lesson Number Date

Section 3.3 Properties of Logarithms

Section Objectives: Students will know how to rewrite logarithmic functions with a different base, use properties of logarithms to evaluate, rewrite, expand, or condense logarithmic expressions.

I. Change of Base (p. 239) Pace: 5 minutes

1. Note that our calculators have only two buttons for logarithmic functions, base 10 and base e . State the **Change-of-Base Formula**.

Let a , b , and x be positive real numbers such that $a \neq 1$ and $b \neq 1$. Then

$$\log_a x = \frac{\log_b x}{\log_b a} = \frac{\log x}{\log a} = \frac{\ln x}{\ln a}.$$

Example 1. Evaluate the following.

- a) $\log_5 18 = \frac{\ln 18}{\ln 5} \approx 1.7959$
b) $\log_2 42 = \frac{\ln 42}{\ln 2} \approx 5.3923$

II. Properties of Logarithms (p. 240) Pace: 10 minutes

- State that logarithms are exponents, since $y = \log_a x$ if and only if $x = a^y$. Thus the following **properties of logarithms** are similar to the exponent properties.

Let a be a positive real number such that $a \neq 1$, let n be a real number, and let u and v be positive real numbers. Then

- $\log_a(uv) = \log_a u + \log_a v$
- $\log_a(u/v) = \log_a u - \log_a v$
- $\log_a u^n = n \log_a u$

- Discuss the *Historical Note* on page 240 of the text.

III. Rewriting Logarithmic Expression (p. 241) Pace: 10 minutes

Example 2. Expand the logarithmic expression.

a)

$$\begin{aligned}\log(2x^3y^4) &= \log 2 + \log x^3 + \log y^4 \\ &= \log 2 + 3 \log x + 4 \log y\end{aligned}$$

b)

$$\begin{aligned}\ln \frac{\sqrt{x+5}}{y^2} &= \ln \sqrt{x+5} - \ln y^2 \\ &= \frac{1}{2} \ln(x+5) - 2 \ln y\end{aligned}$$

Example 3. Condense the logarithmic expression.

$$2 \log x - 3 \log y + \frac{1}{2} \log z = \log x^2 - \log y^3 + \log \sqrt{z}$$

$$\begin{aligned} \text{a)} \quad &= \log \frac{x^2}{y^3} + \log \sqrt{z} \\ &= \log \frac{x^2 \sqrt{z}}{y^3} \end{aligned}$$

$$\begin{aligned} \text{b)} \quad &\frac{1}{3} (2 \ln x - 4 \ln y - \ln(z+2)) = \frac{1}{3} (\ln x^2 - \ln y^4 - \ln(z+2)) \\ &= \frac{1}{3} \left(\ln \frac{x^2}{y^4} - \ln(z+2) \right) \\ &= \frac{1}{3} \left(\ln \frac{x^2}{y^4(z+2)} \right) \\ &= \ln \sqrt[3]{\frac{x^2}{y^4(z+2)}} \end{aligned}$$

IV. Application (p. 242)

Pace: 5 minutes

Example 4. On the Richter scale, the magnitude R of an earthquake of intensity I is given by

$$R = \frac{\ln I - \ln I_0}{\ln 10}$$

where I_0 is the minimum intensity used for comparison. Write this as a single common logarithmic expression.

$$\begin{aligned} R &= \frac{\ln I - \ln I_0}{\ln 10} \\ &= \frac{\frac{\log I}{\log e} - \frac{\log I_0}{\log e}}{\frac{\log 10}{\log e}} \\ &= \frac{\log I - \log I_0}{1} \\ &= \log \frac{I}{I_0} \end{aligned}$$