Chapter 3 Exponential and Logarithmic Functions

Course/Section
Lesson Number
Date

Section 3.4 Exponential and Logarithmic Equations

Section Objectives: Students will know how to solve exponential and logarithmic equations.

I. Introduction (p. 246) Pace: 5 minutes

- Review the following one-to-one and inverse properties, which will be key in solving exponential and logarithmic equations.

  One-to-One Properties
  1. $a^x = a^y$ if and only if $x = y$.
  2. $\log_a x = \log_a y$ if and only if $x = y$.

  Inverse Properties
  1. $\log_a a^x = x$
  2. $a^{\log_a x} = x$

II. Solving Exponential Equations (pp. 247−248) Pace: 15 minutes

- State that two very general keys to solving exponential equations are:
  1. Isolate the exponential expression.
  2. Use the second one-to-one property from above.

  Example 1. Solve each equation and round your answer to three decimal places.

  a)
  $$4e^{2x} = 16 \Rightarrow e^{2x} = 4 \Rightarrow \ln e^{2x} = \ln 4 \Rightarrow 2x = \ln 4$$
  $$x = \frac{\ln 4}{2} \approx 0.693$$

  b)
  $$5e^{x+2} - 8 = 14 \Rightarrow 5e^{x+2} = 22 \Rightarrow e^{x+2} = \frac{22}{5} \Rightarrow \ln e^{x+2} = \ln\frac{22}{5} \Rightarrow$$
  $$x + 2 = \ln\frac{22}{5} \Rightarrow x = \ln\frac{22}{5} - 2 \approx -0.518$$

  c)
  $$2\left(3^x - 1\right) = 10 \Rightarrow 3^x - 1 = 5 \Rightarrow 3^x = 4 \Rightarrow \ln 3^x = \ln 4 \Rightarrow x \ln 3 = \ln 4$$
  $$x = \frac{\ln 4}{\ln 3} \approx 1.262$$

  d)
  $$e^{2x} - e^x - 20 = 0 \Rightarrow \left(e^x + 4\right)\left(e^x - 5\right) = 0$$
  $$e^x + 4 = 0 \Rightarrow e^x = -4 \text{ or } e^x - 5 = 0 \Rightarrow e^x = 5 \Rightarrow x = \ln 5 \approx 1.609$$

III. Solving Logarithmic Equations (pp. 249−250) Pace: 15 minutes

- State that there are two basic ways of solving logarithmic equations.
  1. Isolate the logarithmic expression and then write the equation in equivalent exponential form.
  2. Get a single logarithmic expression with the same base on each side of the equation; then use the one-to-one property.

Example 2. Solve the following logarithmic equations and round your answers to three decimal places.

a) $2 \log x = 5 \Rightarrow \log x = \frac{5}{2} \Rightarrow x = 10^{5/2} \approx 316.228$

b)

$$\ln\sqrt{x+2} = \ln x \Rightarrow \sqrt{x+2} = x \Rightarrow x+2 = x^2 \Rightarrow x^2 - x - 2 = 0$$

$$(x-2)(x+1) = 0$$

$$x - 2 = 0 \Rightarrow x = 2$$

$$x + 1 = 0 \Rightarrow x = -1$$

−1 cannot be a solution because of the domain of the logarithmic function.

c)

$$\log x - \log(x-3) = 1 \Rightarrow \log\frac{x}{x-3} = 1 \Rightarrow \frac{x}{x-3} = 10^1 \Rightarrow$$

$$x = 10x - 30 \Rightarrow -9x = -30 \Rightarrow x = \frac{10}{3}$$

IV. Applications (pp. 251−252) Pace: 10 minutes

Example 3. How long would it take for an investment to double if the interest were compounded continuously at 8%?

$$2P = Pe^{0.08t}$$

$$2 = e^{0.08t}$$

$$\ln 2 = \ln e^{0.08t}$$

$$\ln 2 = 0.08t$$

$$t = \frac{\ln 2}{0.08} \approx 8.66 \text{ years}$$

Example 4. You have $50,000 to invest. You need to have $350,000 to retire in thirty years. At what continuously compounded interest rate would you need to invest to reach your goal?

$$350,000 = 50,000e^{r \cdot 30}$$

$$7 = e^{30r}$$

$$\ln 7 = \ln e^{30r}$$

$$\ln 7 = 30r$$

$$r = \frac{\ln 7}{30} \approx 6.5\%$$