Chapter 3 Exponential and Logarithmic Functions

Course/Section
Lesson Number
Date

Section 3.5 Exponential and Logarithmic Models

Section Objectives: Students will know how to use exponential growth models, exponential decay models, Gaussian models, and logistic growth models to solve applications.

I. Introduction (p. 257) Pace: 5 minutes

- State that there will be five different mathematical models looked at in this section. They are:
  1. **Exponential growth model:** $y = ae^{bx}, b > 0$
  2. **Exponential decay model:** $y = ae^{bx}, b < 0$
  3. **Gaussian model:** $y = ae^{-(x-b)^2/c}$
  4. **Logistic growth model:** $y = \dfrac{a}{1 + be^{-rx}}$
  5. **Logarithmic models:** $y = a + b\ln x$, $y = a + b\log_{10}x$

- Direct the students' attention to the graphs on page 257 of the text.

II. Exponential Growth and Decay (pp. 258−260) Pace: 15 minutes

Example 1. The population of a large city, in millions, can be modeled by $y = 1.8e^{0.026x}$, where $x = 0$ corresponds to 1980. In what year is the population of this city expected to reach 2.5 million?

$$1.8e^{0.026x} = 2.5$$
$$e^{0.026x} = \frac{25}{18}$$
$$\ln e^{0.026x} = \ln\frac{25}{18}$$
$$0.026x = \ln\frac{25}{18}$$
$$x = \frac{\ln\frac{25}{18}}{0.026} \approx 12.6 \text{ years}$$

Example 2. Two hours after bacteria were introduced into a culture, the population was 256. Five hours after that, the population was 541. What will the population be 24 hours after the start of the experiment?

$$
\begin{aligned}
y &= ae^{bx} \\
256 &= ae^{b\cdot 2} \\
a &= 256e^{-2b} \\
541 &= 256e^{-2b}e^{b\cdot 5} = 256e^{3b} \\
e^{3b} &= \frac{541}{256} \\
\ln e^{3b} &= \ln\frac{541}{256} \\
3b &= \ln\frac{541}{256} \\
b &= 3^{-1}\ln\frac{541}{256} \\
a &= 256e^{-2\left(3^{-1}\ln\frac{541}{256}\right)} \\
y &= 256e^{-2\left(3^{-1}\ln\frac{541}{256}\right)}e^{3^{-1}\ln\frac{541}{256}\cdot 24} \approx 61{,}839
\end{aligned}
$$

- Define half-life to be the amount of time required for one-half of an original amount to decay.

Example 3. The radioactive isotope ^{226}Ra has a half-life of 1620 years. If the original amount was 5 grams, how much would remain after 10,000 years?

$$
\begin{aligned}
y &= ae^{bx} \\
y &= 5e^{bx} \\
2.5 &= 5e^{b\cdot 1620} \\
e^{1620b} &= 0.5 \\
\ln e^{1620b} &= \ln 0.5 \\
1620b &= \ln 0.5 \\
b &= \frac{\ln 0.5}{1620} \\
y &= 5e^{\frac{\ln 0.5}{1620}\cdot 10{,}000} \approx .07
\end{aligned}
$$

Tip: Tell the students that in this formula, a is always the initial amount. You may also want to tell them that $b = \ln 0.5 \div$ half-life.

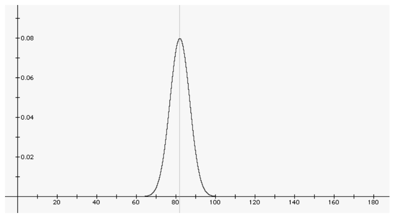
- Discuss the *Technology* on page 258 of the text.

III. Gaussian Models (p. 261) Pace: 5 minutes

Example 4. The test scores at Nick and Tony's Institute of Technology can be modeled by the following normal distribution

$$y = 0.0798e^{-\frac{(x-82)^2}{50}}$$

where x represents the test scores. Sketch the graph and estimate the average test score.



The average score is 82.

IV. Logistic Growth Models (p. 262) Pace: 5 minutes

Example 5. The state game commission released 100 animals into a game preserve. The agency believes that the carrying capacity of the preserve is 1200 animals and that the growth of the herd can be modeled by

$$p(t) = \frac{1200}{1 + 8e^{-0.1588t}}$$

where t is measured in months. How long will it take the herd to reach one half of the preserve's carrying capacity?

$$\frac{1200}{1+8e^{-0.1588t}} = 600$$

$$1+8e^{-0.1588t} = 2$$

$$8e^{-0.1588t} = 1$$

$$e^{-0.1588t} = \frac{1}{8}$$

$$\ln e^{-0.1588t} = \ln\frac{1}{8}$$

$$-0.1588t = \ln\frac{1}{8}$$

$$t = \frac{\ln\frac{1}{8}}{-0.1588} \approx 13 \text{ months}$$

V. **Logarithmic Models** (p. 263) Pace: 5 minutes

Example 6. On the Richter scale, the magnitude R of an earthquake of intensity I is $R = \log(I/I_0)$, where $I_0 = 1$ is the minimum intensity used for comparison. What is the magnitude of an earthquake with intensity 80,000,000?

$$R = \log 80{,}000{,}000 \approx 7.9$$

- Assign the *Writing About Mathematics* on page 263 of the text.