

# Chapter 4 Trigonometry

Course/Section  
Lesson Number  
Date

## Section 4.2 Trigonometric Functions: The Unit Circle

**Section Objectives:** Students will know how to evaluate trigonometric functions.

### I. The Unit Circle (p. 294)

Pace: 5 minutes

- Describe how we can evaluate trigonometric functions for any real number (not just for angles) as follows. Let  $t$  be any real number. Mark off an arc of length  $t$  along the unit circle starting at the point  $(1, 0)$  and moving counterclockwise for positive numbers and clockwise for negative numbers. Let  $\theta$  be the angle in standard position that is subtended by this arc. Then  $s = r\theta = \theta$ .

### II. The Trigonometric Functions (pp. 295–297)

Pace: 10 minutes

- State the following **definitions of trigonometric functions**. Let  $t$  be a real number and let  $(x, y)$  be the point on the unit circle corresponding to  $t$ .

$$\begin{aligned}\sin t &= y & \cos t &= x \\ \tan t &= \frac{y}{x}, x \neq 0 & \cot t &= \frac{x}{y}, y \neq 0 \\ \sec t &= \frac{1}{x}, x \neq 0 & \csc t &= \frac{1}{y}, y \neq 0\end{aligned}$$

**Example 1.** Evaluate the six trigonometric functions at each real number.

a)  $t = \pi/3$

$$(x, y) = \left( \frac{1}{2}, \frac{\sqrt{3}}{2} \right)$$

$$\begin{aligned}\sin \frac{\pi}{3} &= y = \frac{\sqrt{3}}{2} & \csc \frac{\pi}{3} &= \frac{1}{y} = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3} \\ \cos \frac{\pi}{3} &= x = \frac{1}{2} & \sec \frac{\pi}{3} &= \frac{1}{x} = 2 \\ \tan \frac{\pi}{3} &= \frac{y}{x} = \frac{\sqrt{3}/2}{1/2} = \sqrt{3} & \cot \frac{\pi}{3} &= \frac{x}{y} = \frac{1/2}{\sqrt{3}/2} = \frac{\sqrt{3}}{3}\end{aligned}$$

b)  $t = 3\pi/4$

$$(x, y) = \left( -\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2} \right)$$

$$\begin{aligned}\sin \frac{3\pi}{4} &= y = \frac{\sqrt{2}}{2} & \csc \frac{3\pi}{4} &= \frac{1}{y} = \sqrt{2} \\ \cos \frac{3\pi}{4} &= x = -\frac{\sqrt{2}}{2} & \sec \frac{3\pi}{4} &= \frac{1}{x} = -\sqrt{2} \\ \tan \frac{3\pi}{4} &= \frac{y}{x} = -1 & \cot \frac{3\pi}{4} &= \frac{x}{y} = -1\end{aligned}$$

**III. Domain and Period of Sine and Cosine** (pp. 297–298) Pace: 5 minutes

- State that the domain of sine and cosine is  $(-\infty, \infty)$ , and the range is  $[-1, 1]$ .
- Define a function  $f$  to be **periodic with period  $p$**  if  $f$  is a function for which  $p$  is the smallest positive number such that  $f(x + p) = f(x)$  for all  $x$  in the domain of  $f$ . Note that the sine and cosine are periodic with period  $2\pi$ .
- State that cosine and secant are even functions, and the other four are odd functions.

**IV. Evaluating Trigonometric Functions with a Calculator** (p. 298)

Pace: 5 minutes

**Tip:** In trigonometry, an overwhelming number of incorrect answers come from the calculator being in the wrong mode.

**Example 2.** Evaluate the following by using a calculator:

$\csc 5$ .

After the calculator is in radian mode, enter

$\boxed{1} \boxed{\text{SIN}} \boxed{5} \boxed{)} \boxed{\text{x}^{-1}} \boxed{\text{ENTER}}$ .  $\csc 5 \approx -1.04284$