Chapter 4 Trigonometry

Course/Section Lesson Number Date

Section 4.3 Right Angle Trigonometry

Section Objectives: Students will know how to use the fundamental trigonometric identities.

I. The Six Trigonometric Functions (pp. 301−303) Pace: 15 minutes

- Define the six trigonometric functions, **sine, cosine, tangent, cotangent, secant, and cosecant,** as follows. Let θ be an acute angle of a right triangle. Then

$$\sin\theta = \frac{\text{opp}}{\text{hyp}} \quad \cos\theta = \frac{\text{adj}}{\text{hyp}} \quad \tan\theta = \frac{\text{opp}}{\text{adj}}$$
$$\csc\theta = \frac{\text{hyp}}{\text{opp}} \quad \sec\theta = \frac{\text{hyp}}{\text{adj}} \quad \cot\theta = \frac{\text{adj}}{\text{opp}}$$

where opp = the length of the side *opposite* θ
adj = the length of the side *adjacent to* θ
hyp = the length of the *hypotenuse*

Tip: Emphasize that we are shortening the function notation--that is, $\sin\theta$ is really $\sin(\theta)$.

- Draw a 45°-45°-90° triangle, labeling both legs with length 1. Use the Pythagorean Theorem to determine that the length of the hypotenuse is $\sqrt{2}$. This leads to $\sin 45° = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$ $\quad \cos 45° = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$ $\quad \tan 45° = \frac{1}{1} = 1$.

- Now draw a 30°-60°-90° triangle. Remind students that the length of the hypotenuse of such a triangle is twice the length of the shorter leg. Label the hypotenuse and one leg with lengths 2 and 1, respectively. Use the Pythagorean Theorem to determine that the length of the other leg is $\sqrt{3}$. This leads to

$$\sin 30° = \frac{1}{2} \quad \cos 30° = \frac{\sqrt{3}}{2} \quad \tan 30° = \frac{\sqrt{3}}{3}$$
$$\sin 60° = \frac{\sqrt{3}}{2} \quad \cos 60° = \frac{1}{2} \quad \tan 60° = \sqrt{3}$$

Tip: Since the above nine values are encountered so frequently, students should memorize them. The students should also learn to construct the triangles in Figures 4.28 and 4.29.

- Draw attention to the cofunction relationships in the above. Then state

$\sin(90° - \theta) = \cos\theta$ $\qquad$ $\cos(90° - \theta) = \sin\theta$
$\tan(90° - \theta) = \cot\theta$ $\qquad$ $\cot(90° - \theta) = \tan\theta$
$\sec(90° - \theta) = \csc\theta$ $\qquad$ $\csc(90° - \theta) = \sec\theta$

II. Trigonometric Identities (pp. 304−305) Pace: 15 minutes

- State the **fundamental trigonometric identities** in three stages. First, from the original definitions, we have the **reciprocal identities**

$$\sin\theta = \frac{1}{\csc\theta} \quad \cos\theta = \frac{1}{\sec\theta} \quad \tan\theta = \frac{1}{\cot\theta}$$
$$\csc\theta = \frac{1}{\sin\theta} \quad \sec\theta = \frac{1}{\cos\theta} \quad \cot\theta = \frac{1}{\tan\theta}$$

Second, from the original definitions and the reciprocal identities, we have the **quotient identities**

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \quad \cot\theta = \frac{\cos\theta}{\sin\theta}$$

Finally, from the Pythagorean Theorem, $(\text{opp})^2 + (\text{adj})^2 = (\text{hyp})^2$, and dividing both sides of the equation by $(\text{hyp})^2$, we have the **Pythagorean identities**

$$\sin^2\theta + \cos^2\theta = 1 \quad \tan^2\theta + 1 = \sec^2 \quad 1 + \cot^2\theta = \csc^2\theta.$$

Example 1. If θ is an acute angle such that $\cos\theta = 0.3$, find the following.

a) $\sin\theta$

$$\sin^2\theta + \cos^2\theta = 1$$

$$\sin^2\theta + 0.3^2 = 1$$

$$\sin^2\theta = 1 - 0.3^2 = 0.91$$

$$\sin\theta = \sqrt{0.91} = \frac{\sqrt{91}}{10}$$

b) $\tan\theta = \dfrac{\sin\theta}{\cos\theta} = \dfrac{\sqrt{0.91}}{0.3} = \dfrac{\sqrt{91}}{3}$

c) $\cot\theta = \dfrac{1}{\tan\theta} = \dfrac{0.3}{\sqrt{0.91}} = \dfrac{3\sqrt{91}}{91}$

d) $\sec\theta = \dfrac{1}{\cos\theta} = \dfrac{1}{0.3} = \dfrac{10}{3}$

e) $\csc\theta = \dfrac{1}{\sin\theta} = \dfrac{10}{\sqrt{91}} = \dfrac{10\sqrt{91}}{91}$

III. Evaluating Trigonometric Functions with a Calculator (p. 305)
Pace: 5 minutes

Tip: In trigonometry, an overwhelming number of incorrect answers come from the calculator being in the wrong mode.

Example 2. Evaluate the following by using a calculator:
cos 15.3°.
After the calculator is in degree mode, enter
[COS] 15.3 [ENTER] . cos 15.3° ≈ .9646

IV. Applications Involving Right Triangles (pp. 306–307) Pace: 10 minutes

Example 3. If the sun is 30° up from the horizon and shining on a tree forming a 50-foot shadow, how tall is the tree?

$$\frac{h}{50} = \tan 30° \Rightarrow h = 50\cdot\frac{\sqrt{3}}{3} \approx 28.87 \text{ feet}$$

Example 4. If a rope tied to the top of a flagpole is 35 feet long, then what angle is formed by the rope and the ground when the rope is pulled to the ground, 25 feet from the base of the pole?
$\cos\theta = 25/35 \Rightarrow \theta \approx 44.42°$.