Chapter 4 Trigonometry

Course/Section
Lesson Number
Date

Section 4.4 Trigonometric Functions of Any Angle

Section Objectives: Students will know how to evaluate trigonometric functions of any angle and of real numbers.

I. Introduction (pp. 312−313) Pace: 10 minutes

- State the following **definitions of trigonometric functions of any angle.** Let θ be an angle in standard position with (x, y) a point on the terminal side of θ and $r = \sqrt{x^2 + y^2} \neq 0$.

$$\sin\theta = \frac{y}{r} \qquad \cos\theta = \frac{x}{r}$$
$$\tan\theta = \frac{y}{x},\ x \neq 0 \qquad \cot\theta = \frac{x}{y},\ y \neq 0$$
$$\sec\theta = \frac{r}{x},\ x \neq 0 \qquad \csc\theta = \frac{r}{y},\ y \neq 0$$

Example 1. Let (4, −3) be on the terminal side of θ. Find the value of the sine, cosine, and tangent of θ.
$r = \sqrt{4^2 + (-3)^2} = 5$. So, $\sin\theta = -3/5$, $\cos\theta = 4/5$, and $\tan\theta = -3/4$.

- Discuss the signs of the trigonometric functions in the four quadrants. Because both x and y are positive in the first quadrant, all six functions are positive in the first quadrant. Because only y is positive in the second quadrant, only sine and cosecant are positive in the second quadrant. Since both x and y are negative in the third quadrant, only tangent and cotangent are positive in the third quadrant. Since only x is positive in the fourth quadrant, only cosine and secant are positive in the fourth quadrant.

Example 2. Given $\cos\theta = 3/5$ and $\tan\theta < 0$, find $\sin\theta$ and $\cot\theta$.
θ must be in the fourth quadrant. Using $x = 3$, $y = -4$, and $r = 5$, we have $\sin\theta = -4/5$ and $\cot\theta = -3/4$.

II. Reference Angles (p. 314) Pace: 5 minutes

- State that if θ is in standard position, then the **reference angle** θ' associated with θ is the acute angle formed by the terminal side of θ and the x-axis.

Tip: Emphasize that the reference angle uses only the x-axis.

Example 3. Find the reference angle for the following angles.

a) $\theta = 125°$
$\theta' = 180° - 125° = 55°$

b) $\theta = 5$
$\theta' = 2\pi - 5 \approx 1.2832$

III. Trigonometric Functions of Real Numbers (pp. 315−317)
Pace:15 minutes

- State that if θ is in standard position with (x, y) on its terminal side, then if θ' is placed in standard position, ($|x|$, $|y|$) will be on its terminal side. Also, the r for θ and the r for θ' will be the same. Hence, $|\sin\theta| = \sin\theta'$, $|\cos\theta| = \cos\theta'$, and $|\tan\theta| = \tan\theta'$. This means that to evaluate trigonometric functions of any angle, we need only find the value of that function for the reference angle and attach the proper sign, according to the quadrant in which θ lies.

- Tell the students that they should memorize the table on page 315 of the text.

Example 4. Evaluate the following trigonometric functions.

a) sin 210°. Since 210° is in quadrant III and the reference angle is 30°, sin 210° = −sin 30° = 1/2.

b) cos (−5π/4). Since −5π/4 is in the second quadrant and the reference angle is π/4, $\cos\left(-\frac{5\pi}{4}\right) = -\cos\frac{\pi}{4} = -\frac{\sqrt{2}}{2}$.

c) tan 5π/3. Since 5π/3 is in the fourth quadrant and the reference angle is π/3, $\tan\frac{5\pi}{3} = -\tan\frac{\pi}{3} = -\sqrt{3}$.

Example 5. Let θ be an angle in the third quadrant such that $\cos\theta$ = −1/4. Find the following.

a) $\sin\theta$

$$\sin^2\theta + \left(-\frac{1}{4}\right)^2 = 1$$

$$\sin^2\theta = \frac{15}{16}$$

$$\sin\theta = -\frac{\sqrt{15}}{4}$$

b) $\tan\theta$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{-\sqrt{15}/4}{-1/4} = \sqrt{15}$$

Example 6. Use a calculator to solve $\sin\theta = 0.2358$.

SIN⁻¹ .2358 ENTER . $\theta \approx 0.2380$