Chapter 4 Trigonometry

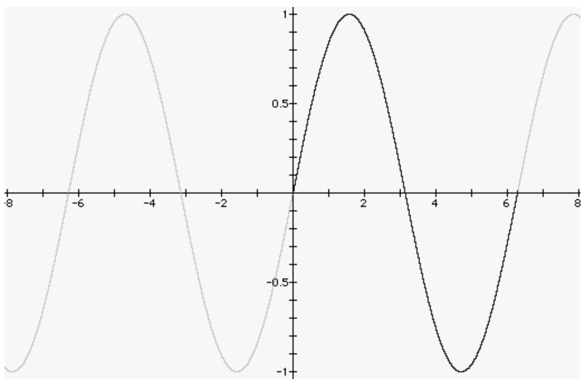
Course/Section
Lesson Number
Date

Section 4.5 Graphs of Sine and Cosine Functions

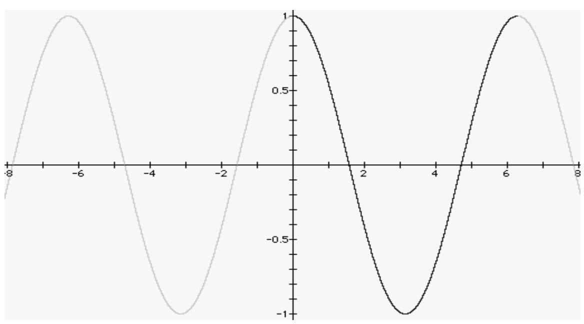
Section Objectives: Students will know how to sketch and translate the graphs of sine and cosine functions.

I. Basic Sine and Cosine Curves (pp. 321−322) Pace: 10 minutes

- Discuss the graphs below, noting that the darker portions represent one period.
  $y = \sin x$



$y = \cos x$



- Discuss the **five key points** of both graphs.

	maximums	*minimums*	*intercepts*
$y = \sin x$	$(\pi/2, 1)$	$(3\pi/2, -1)$	$(0, 0)$ $(\pi, 0)$ $(2\pi, 0)$
$y = \cos x$	$(0, 1)$ $(2\pi, 1)$	$(\pi, -1)$	$(\pi/2, 0)$ $(3\pi/2, 0)$

- Discuss the *Technology* feature on page 322 of the text.

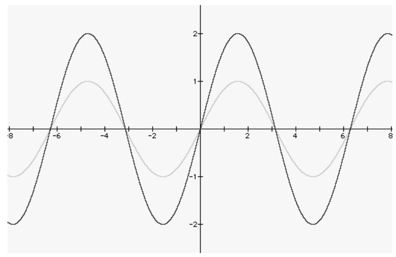
II. Amplitude and Period (pp. 323−324) Pace: 10 minutes

- State that for the rest of the section we apply the material from Section 1.7 to the trigonometric functions. We start by stating that the **amplitude**, or maximum displacement from equilibrium, of $y = a\sin x$ and $y = a\cos x$ is $|a|$.

Example 1. Sketch the graph of the following functions.

a) $y = 2\sin x$

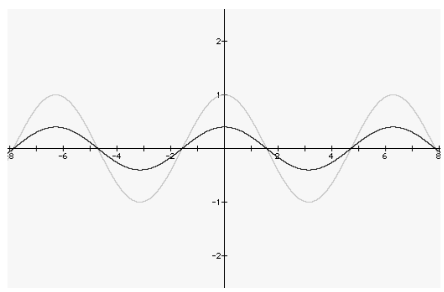
The five key points of this function are (0, 0), (π/2, 2), (π, 0), (3π/2, −2), and (2π, 0).



Tip: Provide the graph of $y = \sin x$ in addition to $y = 2\sin x$.

b) $y = 0.4\cos x$

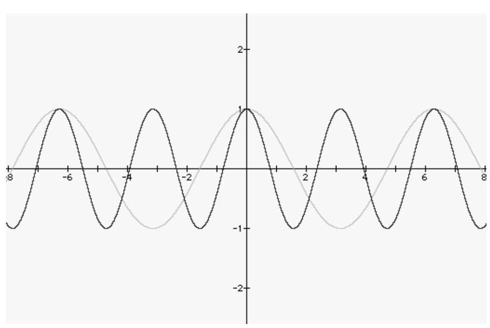
The five key points of this function are (0, 0.4), (π/2, 0), (π,−0.4), (3π/2, 0), and (2π, 0.4).



- State that because $y = a\sin x$ completes one period from $x = 0$ to $x = 2\pi$, it follows that $y = a\sin bx$ completes one period from $x = 0/b = 0$ to $x = 2\pi/b$. Hence the period of both $y = a\sin bx$ and $y = a\cos bx$ is $2\pi/b$.

Example 2. Sketch the graph of $y = \cos 2x$.

The five key points for this function are (0, 1), (π/4, 0), (π/2, −1), (3π/4, 0), and (π, 1).

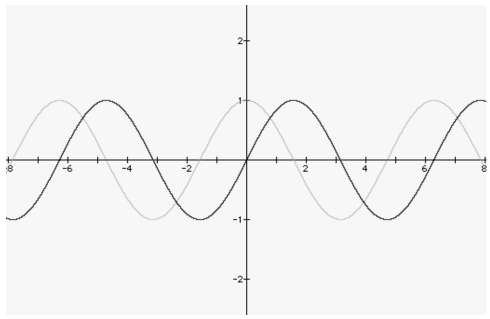


III. Translations of Sine and Cosine Curves (pp. 325−326) Pace: 10 minutes

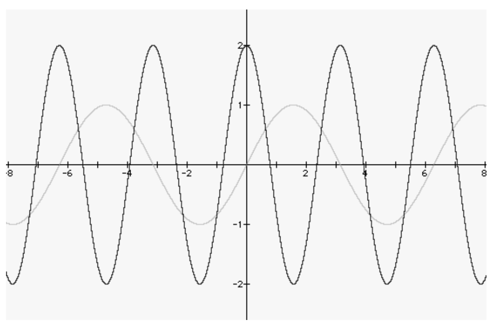
- State that the graphs of $y = a\sin(bx - c) = a\sin b(x - c/b)$ and $y = a\cos(bx - c) = a\cos b(x - c/b)$ have a directed horizontal shift of c/b.

Example 3. Sketch the graph of the following.

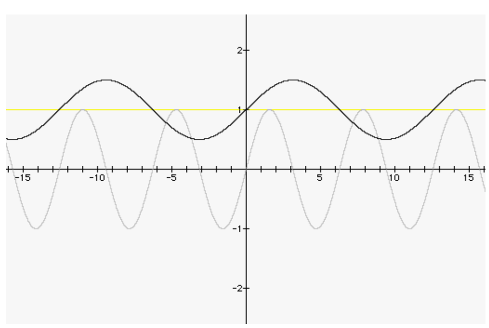
a) $y = \cos(x - \pi/2)$
The five key points for this function are $(\pi/2, 1)$, $(\pi, 0)$, $(3\pi/2, -1)$, $(2\pi, 0)$, and $(5\pi/2, 1)$.



b) $y = 2\sin(2x + \pi/2)$
The five key points for this function are $(-\pi/4, 0)$, $(0, 2)$, $(\pi/4, 0)$, $(\pi/2, -2)$, and $(3\pi/4, 0)$.



c) $y = 1 - 0.5\sin(0.5x - \pi)$
Note that this graph has an upward vertical shift of 1 unit. Also, the horizontal shift is 2π, giving the appearance of no horizontal shift. The five key points for this function are $(0, 1)$, $(\pi, 3/2)$, $(2\pi, 1)$, $(3\pi, 1/2)$, and $(4\pi, 1)$.



IV. Mathematical Modeling (p. 327) Pace: 5 minutes

Example 4. The average monthly temperatures of a certain southern city can be modeled by

$$T = 74.6 + 12.87\sin(0.52t - 2.09)$$

where T is the average monthly temperature in degrees and t is the month, with $t = 1$ corresponding to January. Use this model to predict the average monthly temperature in June.

Evaluate at $t = 6$ to get $T \approx 85.63°$.