# Chapter 4 Trigonometry

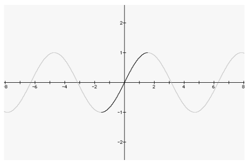
Course/Section
Lesson Number
Date

## Section 4.7 Inverse Trigonometric Functions

**Section Objectives:** Students will know how to evaluate the inverse trigonometric functions and compositions of trigonometric functions

**I. Inverse Sine Function** (pp. 343−344) Pace: 10 minutes

- State that because the sine function does not pass the Horizontal Line Test, we must restrict its domain in order for its inverse to be a function. We restrict the domain to $[-\pi/2, \pi/2]$. Draw the graph below.



- State that the **inverse sine function** can be denoted by $y = \sin^{-1}x$ or by $y = \arcsin x$. It can be thought of as the angle whose sine is $x$.

**Example 1.** Evaluate the following.

**a)** $\sin(-1) = -\pi/2$

**b)** $\arcsin(1/2) = \pi/6$

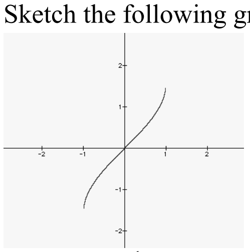
**II. Other Inverse Trigonometric Functions** (pp. 345−346) Pace: 10 minutes

- State that the domain of the cosine function must be restricted to $[0, \pi]$ and the domain of the tangent function to $(-\pi/2, \pi/2)$, so that their inverses will be functions.

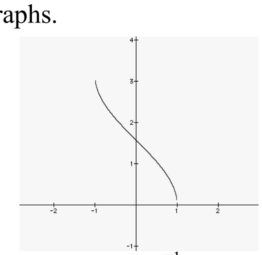
- State the following definitions.

| *Function* | *Domain* | *Range* |
|---|---|---|
| $y = \sin^{-1}x \Leftrightarrow \sin y = x$ | $[-1, 1]$ | $[-\pi/2, \pi/2]$ |
| $y = \cos^{-1}x \Leftrightarrow \cos y = x$ | $[-1, 1]$ | $[0, \pi]$ |
| $y = \tan^{-1}x \Leftrightarrow \tan y = x$ | $(-\infty, \infty)$ | $(-\pi/2, \pi/2)$ |

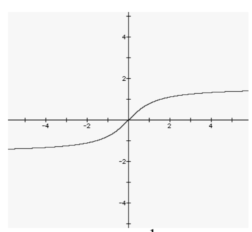
- Sketch the following graphs.



$y = \sin^{-1} x$



$y = \cos^{-1}x$



$y = \tan^{-1}x$

**Example 2.** Evaluate the following.

**a)** $\cos^{-1}\dfrac{\sqrt{3}}{2} = \dfrac{\pi}{6}$

**b)** $\tan^{-1} 1 = \pi/4$

**Example 3.** Use a calculator to evaluate the following.

**a)** $\sin^{-1} 0.5524$

After the calculator is in radian mode, use the following keystrokes. SIN⁻¹ .5524 ENTER . $\sin^{-1} 0.5524 \approx 0.5852$

**b)** $\tan^{-1}(-3.254)$

After the calculator is in radian mode, use the following keystrokes. TAN⁻¹ (−) 3.254 ENTER .

$\tan^{-1}(-3.254) \approx -1.2726$

**III. Compositions of Functions** (pp. 347−348) Pace: 10 minutes

- Review, from Section 1.9, that for inverse functions, $f(f^{-1}(x)) = x$ and $f^{-1}(f(x)) = x$.

- State the **inverse properties of trigonometric functions**.
  1. If $-1 \le x \le 1$ and $-\pi/2 \le y \le \pi/2$, then $\sin(\sin^{-1} x) = x$ and $\sin^{-1}(\sin y) = y$.
  2. If $-1 \le x \le 1$ and $0 \le y \le \pi$, then $\cos(\cos^{-1} x) = x$ and $\cos^{-1}(\cos y) = y$.
  3. If $-\pi/2 < y < \pi/2$, then $\tan(\tan^{-1} x) = x$ and $\tan^{-1}(\tan y) = y$.

**Example 4.** Evaluate the following.

**a)** $\sin(\arcsin 0.12) = 0.12$

**b)** $\arctan(\tan 5\pi/6) = \arctan[\tan(-\pi/6)] = -\pi/6$

**c)** $\cos(\arccos 6)$. 6 is not in the domain of the inverse cosine.

**Example 5.** Find the exact value.

**a)** $\cos(\sin^{-1} 3/5)$

Let $u = \sin^{-1} 3/5$ and draw a right triangle showing $u$, with the side opposite $u$ having length 3 and the hypotenuse having length 5. Use the Pythagorean Theorem to label the other side 4. Then $\cos(\sin^{-1} 3/5) = \cos u = 4/5$.

**b)** $\sin[\tan^{-1}(-1/2)]$

Let $u = \tan^{-1}(-1/2)$ and draw a right triangle showing $u$, with the side opposite $u$ having length $-1$ and the side adjacent having length 2. Use the Pythagorean Theorem to label the hypotenuse $\sqrt{5}$. Then $\sin[\tan^{-1}(-1/2)] = \sin u = \dfrac{-1}{\sqrt{5}}$.

**Example 6.** Write $\sin(\cos^{-1} x)$ as an algebraic expression in $x$.

Let $u = \cos^{-1} x$ and draw a right triangle showing $u$, with the side adjacent $u$ having length $x$ and the hypotenuse having length 1. Use the Pythagorean Theorem to label the other side $\sqrt{1 - x^2}$.

Then $\sin(\cos^{-1} x) = \sin u = \sqrt{1 - x^2}/1 = \sqrt{1 - x^2}$.