Chapter 5 Analytic Trigonometry

Section 5.1 Using Fundamental Identities

Course/Section
Lesson Number
Date

Section Objectives: Students will know how to use fundamental trigonometric identities to evaluate trigonometric functions and simplify trigonometric expressions.

I. Introduction (p. 374) Pace: 5 minutes

- Review the following list of identities that we have covered so far.

Reciprocal Identities

$\sin u = \frac{1}{\csc u}$ $\cos u = \frac{1}{\sec u}$ $\tan u = \frac{1}{\cot u}$

$\csc u = \frac{1}{\sin u}$ $\sec u = \frac{1}{\cos u}$ $\cot u = \frac{1}{\tan u}$

Quotient Identities

$\tan u = \frac{\sin u}{\cos u}$ $\cot u = \frac{\cos u}{\sin u}$

Pythagorean Identities

$\sin^2 u + \cos^2 u = 1$ $\tan^2 u + 1 = \sec^2 u$ $1 + \cot^2 u = \csc^2 u$

Cofunction Identities

$\sin(90° - u) = \cos u$ $\cos(90° - u) = \sin u$
$\tan(90° - u) = \cot u$ $\cot(90° - u) = \tan u$
$\sec(90° - u) = \csc u$ $\csc(90° - u) = \sec u$

Even/Odd Identities

$\sin(-u) = -\sin u$ $\tan(-u) = -\tan u$ $\sec(-u) = \sec u$
$\cos(-u) = \cos u$ $\cot(-u) = -\cot u$ $\csc(-u) = -\csc u$

Tip: State that for each one of these, there are two other versions that the students need to be familiar with.

II. Using the Fundamental Identities (pp. 375–378) Pace: 20 minutes

Example 1. If $\csc u = -5/3$ and $\cos u > 0$, find the values of the other five trigonometric functions.

$\sin u = -3/5$

$\cos^2 u = 1 - \sin^2 u = 1 - \left(-\frac{3}{5}\right)^2 = \frac{16}{25}$

$\cos u = \frac{4}{5}$

$\sec u = 5/4$

$\tan u = \frac{\sin u}{\cos u} = \frac{-3/5}{4/5} = -\frac{3}{4}$

$\cot u = -4/3$

Example 2. Simplify the following.

a) $\csc^2 x \cot x - \cot x$
$= (\csc^2 x - 1)\cot x = (\cot^2 x)\cot x = \cot^3 x$

b) $\tan x \sin x + \cos x$
$= \left(\frac{\sin x}{\cos x}\right)\sin x + \cos x = \frac{\sin^2 x + \cos^2 x}{\cos x} = \frac{1}{\cos x} = \sec x$

c)

$$\frac{\sec t}{\tan t} - \frac{\tan t}{1 + \sec t}$$

$$= \frac{\sec t(1 + \sec t) - \tan^2 t}{\tan t(1 + \sec t)} = \frac{\sec t + \sec^2 t - \tan^2 t}{\tan t(1 + \sec t)}$$

$$= \frac{\sec t + 1}{\tan t(1 + \sec t)} = \frac{1}{\tan t} = \cot t$$

Example 3. Factor the following trigonometric expressions.

a) $\cos^2 x - 1$
$= (\cos x + 1)(\cos x - 1)$

b) $\sin^2 u - 3\sin u - 10$
$= (\sin u + 2)(\sin u - 5)$

c) $\sec^2 t - \tan t - 3$
$= (\tan^2 t + 1) - \tan t - 3 = \tan^2 t - \tan t - 2$
$= (\tan t + 1)(\tan t - 2)$

Example 4. Rewrite $\frac{1}{\sec x - 1}$ so that it is not a fraction.

$$\frac{1}{\sec x - 1} = \frac{1}{\sec x - 1} \cdot \frac{\sec x + 1}{\sec x + 1}$$

$$= \frac{\sec x + 1}{\sec^2 x - 1}$$

$$= \frac{\sec x + 1}{\tan^2 x}$$

$$= \frac{\sec x}{\tan^2 x} + \frac{1}{\tan^2 x}$$

$$= \cot x \csc x + \cot^2 x$$

Example 5. Use the substitution $x = 3\sin u$, $0 < u < \pi/2$, to express $\sqrt{9 - x^2}$ as a function of u.

$$\sqrt{9 - x^2} = \sqrt{9 - (3\sin u)^2}$$

$$= \sqrt{9 - 9\sin^2 u}$$

$$= 3\sqrt{1 - \sin^2 u}$$

$$= 3\sqrt{\cos^2 u}$$

$$= 3\cos u$$