Chapter 5 Analytic Trigonometry

Course/Section Lesson Number Date

Section 5.2 Verifying Trigonometric Identities

Section Objectives: Students will know how to verify trigonometric identities.

I. Introduction (p. 382) Pace: 5 minutes

- State that we will solve conditional equations and verify identities.

- State that when verifying identities, there are no set techniques that you apply every time. We supply the following guidelines, which can be found on page 382 of the text, as just that – guidelines.
  1. Work with only one side of the equation at a time. Usually it is better to start with the more complicated side first.
  2. Look for opportunities to factor an expression, add fractions, square a two-term quantity, or create a single-term denominator.
  3. Look for opportunities to use the fundamental identities. Note which functions are in the final expression you want. Sine and cosine pair well, as do secant and tangent, and cosecant and cotangent.
  4. As a last resort, convert all terms to sine and cosine.
  5. Always try something. Even paths that lead to dead ends give you insight.

II. Verifying Trigonometric Identities (pp. 382−386) Pace: 30 minutes

Example 1. Verify the identity.

$$\frac{\sin^2 x - 1}{\sin^2 x} = -\cot^2 x$$

$$\frac{\sin^2 x - 1}{\sin^2 x} = \frac{\sin^2 x}{\sin^2 x} - \frac{1}{\sin^2 x} = 1 - \csc^2 x = 1 - \left(1 + \cot^2 x\right) = -\cot^2 x$$

Example 2. Verify the identity.

$$\frac{1}{\sec x - 1} - \frac{1}{\sec x + 1} = 2\cot^2 x$$

$$\frac{1}{\sec x - 1} - \frac{1}{\sec x + 1} = \frac{\sec x + 1 - (\sec x - 1)}{\sec^2 x - 1}$$

$$= \frac{2}{\tan^2 x}$$

$$= 2\cot^2 x$$

Example 3. Verify the identity.

$$\left(1 + \cot^2 x\right)\left(1 - \sin^2 x\right) = \cot^2 x$$

$$\left(1 + \cot^2 x\right)\left(1 - \sin^2 x\right) = \csc^2 x \cos^2 x$$

$$= \frac{\cos^2 x}{\sin^2 x}$$

$$= \cot^2 x$$

Example 4. Verify the identity.

$$\sec u + \tan t = \frac{1}{\sec u - \tan u}$$

$$\begin{aligned}
\sec u + \tan t &= \frac{1}{\cos u} + \frac{\sin u}{\cos u} \\
&= \frac{1+\sin u}{\cos u} \cdot \frac{1-\sin u}{1-\sin u} \\
&= \frac{1-\sin^2 u}{\cos u(1-\sin u)} \\
&= \frac{\cos^2 u}{\cos u(1-\sin u)} \\
&= \frac{\cos u}{1-\sin u} \cdot \frac{\sec u}{\sec u} \\
&= \frac{1}{\sec u - \tan u}
\end{aligned}$$

Example 5. Verify the identity.

$$\cot t \cos t = \csc t - \sin t$$

$$\begin{aligned}
\cot t \cos t &= \frac{1}{\tan t \sec t} \\
&= \frac{\sec^2 t - \tan^2 t}{\tan t \sec t} \\
&= \frac{\sec^2 t}{\tan t \sec t} - \frac{\tan^2 t}{\tan t \sec t} \\
&= \frac{\sec t}{\tan t} - \frac{\tan t}{\sec t} \\
&= \frac{1}{\cos t} \cdot \frac{\cos t}{\sin t} - \frac{\sin t}{\cos t} \cdot \cos t \\
&= \csc t - \sin t
\end{aligned}$$

Example 6. Verify the identity.

$$\frac{\cot^2 \theta}{1+\csc\theta} = \frac{1-\sin\theta}{\sin\theta}$$

$$\begin{aligned}
\frac{\cot^2 \theta}{1+\csc\theta} &= \frac{\csc^2\theta - 1}{1+\csc\theta} \\
&= \frac{(\csc\theta - 1)(\csc\theta + 1)}{1+\csc\theta} \\
&= \csc\theta - 1 \\
&= \frac{1}{\sin\theta} - 1 \\
&= \frac{1-\sin\theta}{\sin\theta}
\end{aligned}$$

Tip: This was Example 6 from the text, though we worked with only one side. You should never work with both sides of an identity.