Chapter 5 Analytic Trigonometry

Course/Section
Lesson Number
Date

Section 5.3 Solving Trigonometric Equations

Section Objectives: Students will know how to use standard algebraic techniques and inverse trigonometric functions to solve trigonometric equations.

I. Introduction (pp. 389−391) Pace: 15 minutes

- State that the preliminary goal in solving trigonometric equations is to isolate the trigonometric function involved in the equation.

Example 1. Solve $1 - 2\cos x = 0$.

$1 - 2\cos x = 0$

$\cos x = 1/2$

$x = \pi/3 + 2\pi n$ or $x = 5\pi/3 + 2\pi n$

Example 2. Solve $\sin x + 1 = -\sin x$.

$\sin x + 1 = -\sin x$

$2\sin x + 1 = 0$

$\sin x = -1/2$

$x = 7\pi/6 + 2\pi n$ or $x = 11\pi/6 + 2\pi n$

Example 3. Solve $\tan^2 x - 3 = 0$

$\tan^2 x - 3 = 0$

$\tan^2 x = 3$

$\tan x = \pm\sqrt{3}$

$x = \dfrac{\pi}{3} + \pi n$ or $x = \dfrac{2\pi}{3} + \pi n$

Example 4. Solve $\sec x \csc x = \csc x$

$\sec x \csc x = \csc x$

$\sec x \csc x - \csc x = 0$

$\csc x(\sec x - 1) = 0$

$\csc x = 0$ or $\sec x - 1 = 0$

$\sec x = 1$

$x = 2\pi n$

II. Equations of Quadratic Type (pp. 391−393) Pace: 15 minutes

Example 5. Solve the following on the interval $[0, 2\pi)$.

a) $2\cos^2 x + \cos x - 1 = 0$

$(2\cos x - 1)(\cos x + 1) = 0$

$2\cos x - 1 = 0$ or $\cos x + 1 = 0$

$\cos x = 1/2$ or $\cos x = -1$

$x = \pi/3, 5\pi/3$ or $x = \pi$

b) $2\cos^2 x + 3\sin x - 3 = 0$

$2(1 - \sin^2 x) + 3\sin x - 3 = 0$

$2\sin^2 x - 3\sin x + 1 = 0$

$(2\sin x - 1)(\sin x - 1) = 0$

$2\sin x - 1 = 0$ or $\sin x - 1 = 0$

$\sin x = 1/2$ or $\sin x = 1$

$x = \pi/6, 5\pi/6$, or $\pi/2$

c) $\sec x + 1 = \tan x$

$(\sec x + 1)^2 = \tan^2 x$

$\sec^2 x + 2\sec x + 1 = \sec^2 x - 1$

$2\sec x = -2$

$\sec x = -1$

$x = \pi$

III. Functions Involving Multiple Angles (p. 394) Pace: 10 minutes

Example 6. Solve the following on the interval $[0, 2\pi)$.

a) $2\sin 2t + 1 = 0$

$\sin 2t = -1/2$

$2t = 4\pi/3, 5\pi/3, 10\pi/3, 11\pi/3$

$t = 2\pi/3, 5\pi/6, 5\pi/3, 11\pi/6$

Tip: Note that since $0 \le t \le 2\pi$, $0 \le 2t \le 4\pi$

b) $\cot (x/2) + 1 = 0$

$\cot (x/2) = -1$

$x/2 = 3\pi/4$

$x = 3\pi/2$

Tip: Note that since $0 \le x \le 2\pi$, $0 \le x/2 \le \pi$

IV. Using Inverse Functions (p. 395) Pace: 5 minutes

Example 7. Solve $\sec^2 x - 3\sec x - 10 = 0$.

$\sec^2 x - 3\sec x - 10 = 0$

$(\sec x - 5)(\sec x + 2) = 0$

$\sec x - 5 = 0$ or $\sec x + 2 = 0$

$\sec x = 5$ or $\sec x = -2$

$x = \sec^{-1} 5 + 2\pi n, 2\pi - \sec^{-1} 5 + 2\pi n$ or

$x = \sec^{-1} (-2) + 2\pi n, -\sec^{-1} (-2) + 2\pi n$

- Assign the *Writing About Mathematics* on page 395 of the text.