Chapter 5 Analytic Trigonometry

Course/Section
Lesson Number
Date

Section 5.4 Sum and Difference Formulas

Section Objectives: Students will know how to use sum and difference formulas to evaluate trigonometric functions, verify identities, and solve trigonometric equations.

I. Using Sum and Difference Formulas (pp. 400−403) Pace: 30 minutes

- State the following **sum and difference formulas**.
  $\sin(u + v) = \sin u \cos v + \cos u \sin v$
  $\sin(u - v) = \sin u \cos v - \cos u \sin v$
  $\cos(u + v) = \cos u \cos v - \sin u \sin v$
  $\cos(u - v) = \cos u \cos v + \sin u \sin v$
  $\tan(u+v) = \dfrac{\tan u + \tan v}{1 - \tan u \tan v}$
  $\tan(u-v) = \dfrac{\tan u - \tan v}{1 + \tan u \tan v}$

Tip: Refer to page 424 of the text for proofs of the cosine and tangent formulas. The sine proofs come from applying the cofunction identities.

Example 1. Find the exact value of sin 15°.
$\sin 15° = \sin(45° - 30°) = \sin 45° \cos 30° - \cos 45° \sin 30°$
$= \dfrac{\sqrt{2}}{2} \cdot \dfrac{\sqrt{3}}{2} - \dfrac{\sqrt{2}}{2} \cdot \dfrac{1}{2} = \dfrac{\sqrt{6} - \sqrt{2}}{4}$

Example 2. Find the exact value of cos 7π/12.
$\cos 7\pi/12 = \cos(\pi/3 + \pi/4) = \cos \pi/3 \cos \pi/4 - \sin \pi/3 \sin \pi/4$
$= \dfrac{1}{2} \cdot \dfrac{\sqrt{2}}{2} - \dfrac{\sqrt{3}}{2} \cdot \dfrac{\sqrt{2}}{2} = \dfrac{\sqrt{2} - \sqrt{6}}{4}$

Example 3. Find the exact value of cos 78° cos 18° + sin 78° sin 18°.
$\cos 78° \cos 18° + \sin 78° \sin 18° = \cos(78° - 18°) = \cos 60° = 1/2.$

Example 4. Write $\tan(\tan^{-1} -1 - \tan^{-1} x)$ as an algebraic expression.
$$\tan\left(\tan^{-1}(-1) - \tan^{-1} x\right) = \frac{\tan\left(\tan^{-1}(-1)\right) - \tan\left(\tan^{-1} x\right)}{1 + \tan\left(\tan^{-1}(-1)\right)\tan\left(\tan^{-1} x\right)}$$
$$= \frac{-1 - x}{1 - (-1)x} = \frac{-1 - x}{1 + x}$$

Example 5. Verify the cofunction identity $\sin(90° - x) = \cos x$.
$\sin(90° - x) = \sin 90° \cos x - \cos 90° \sin x$
$= 1 \bullet \cos x - 0 \bullet \sin x = \cos x - 0 = \cos x$

Example 6. Simplify $\sin(x + 3\pi)$.
$\sin(x + 3\pi) = \sin x \cos 3\pi + \cos x \sin 3\pi = \sin x \bullet (-1) + \cos x \bullet 0$
$= -\sin x$

Example 7. Verify the identity $\sin(x + y)\sin(x - y) = \cos^2 y - \cos^2 x$.
$\sin(x + y)\sin(x - y)$
$= (\sin x \cos y + \cos x \sin y)(\sin x \cos y - \cos x \sin y)$
$= \sin^2 x \cos^2 y - \cos^2 x \sin^2 y$
$= (1 - \cos^2 x)\cos^2 y - \cos^2 x(1 - \cos^2 y)$
$= \cos^2 y - \cos^2 x \;\; \cos^2 y - \cos^2 x + \cos^2 x \cos^2 y$
$= \cos^2 y - \cos^2 x$

Example 8. Solve $\cos(x + \pi/4) - \cos(x - \pi/4) = 1$ in the interval $[0, 2\pi)$.

$$\cos\left(x + \frac{\pi}{4}\right) - \cos\left(x - \frac{\pi}{4}\right) = 1$$

$$\cos x \cos\frac{\pi}{4} - \sin x \sin\frac{\pi}{4} - \left(\cos x \cos\frac{\pi}{4} + \sin x \sin\frac{\pi}{4}\right) = 1$$

$$-2 \sin x \cdot \frac{\sqrt{2}}{2} = 1$$

$$\sin x = -\frac{\sqrt{2}}{2}$$

$$x = \frac{5\pi}{4} \text{ or } x = \frac{7\pi}{4}$$