Chapter 6 Additional Topics in Trigonometry

Course/Section Lesson Number Date

Section 6.1 Law of Sines

Section Objectives: Students will know how to use the Law of Sines to solve and find the areas of oblique triangles.

I. Introduction (pp. 430−431) Pace: 10 minutes

- State that to solve an oblique triangle, we need to be given at least one side and then any other two parts of the triangle.

- State the **Law of Sines.** For any triangle *ABC* with opposite sides *a*, *b*, and *c*,

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}, \text{ or } \frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}.$$

Example 1. Given $A = 123°$, $B = 41°$, and $a = 10$ inches, find *c*.
$C = 180° - 123° - 41° = 16°$

$$\frac{c}{\sin 16^\circ} = \frac{10}{\sin 123^\circ}$$

$$c = \frac{10 \sin 16^\circ}{\sin 123^\circ} \approx 3.29$$

Example 2. A triangular plot of land has interior angles $A = 95°$ and $C = 68°$. If the side between these angles is 115 yards long, what are the lengths of the other two sides?
$b = 115$ and $B = 180° - 95° - 68° = 17°$

$$\frac{a}{\sin 95^\circ} = \frac{115}{\sin 17^\circ}$$

$$a = \frac{115 \sin 95^\circ}{\sin 17^\circ} \approx 391.84 \text{ yds.}$$

$$\frac{c}{\sin 68^\circ} = \frac{115}{\sin 17^\circ}$$

$$c = \frac{115 \sin 68^\circ}{\sin 17^\circ} \approx 364.69 \text{ yds.}$$

II. The Ambiguous Case (SSA) (pp. 432−433) Pace: 20 minutes

- Refer to the triangles on page 432 of the text and discuss the various possibilities listed. Emphasize that in the case in which the given angle is acute, the height of the triangle should be calculated. If the height is less than the length of the opposite side, which is less than the length of the adjacent side, then two triangles are possible.

Example 3. Given $A = 26°$, $b = 5$ feet, and $a = 21$ feet, find the other side and the two other angles of the triangle.
Because $a > b$, there will be one triangle.

$$\frac{\sin B}{5} = \frac{\sin 26^\circ}{21} \Rightarrow B = \sin^{-1}\left(\frac{5 \sin 26^\circ}{21}\right) \approx 5.99^\circ$$

$$C \approx 180^\circ - 26^\circ - 5.99^\circ = 148.01^\circ$$

$$\frac{c}{\sin 148.01^\circ} = \frac{21}{\sin 26^\circ} \Rightarrow c = \frac{21 \sin 148.01^\circ}{\sin 26^\circ} \approx 25.38 \text{ feet}$$

Example 4. Given $B = 78°$, $c = 12$, and $b = 5$, find angle C.

$$\frac{\sin C}{12} = \frac{\sin 78^\circ}{5}$$

$$\sin C = \frac{12 \sin 78^\circ}{5} \approx 2.35 > 1$$

Therefore, there is no triangle.

Example 5. Given $A = 29°$, $a = 6$, and $b = 10$, find B.
Since $h = 10 \sin 29° \approx 4.85 < 6 < 10$, there are two triangles and hence two values for B, B_1 and B_2, which are supplementary.

$$\frac{\sin B_1}{10} = \frac{\sin 29^\circ}{6}$$

$$B_1 = \sin^{-1} \frac{10 \sin 29^\circ}{6} \approx 53.9^\circ$$

$B_2 \approx 180° - 53.9° = 126.1°$

III. Area of an Oblique Triangle (p. 434) Pace: 5 minutes

- State that from the law of sines, the area of a triangle is
  Area = (1/2) (base) (height) = $(1/2)bc \sin A = (1/2)ab \sin C = (1/2)ac \sin B$.

Example 6. Find the area of the triangle for which $B = 143°$, $a = 7$ meters, and $b = 18$ meters.
Area = (1/2)(7)(18)sin 143° ≈ 37.91 m^2

IV. Application (p. 435) Pace: 5 minutes

Example 7. Two fire ranger towers lie on the east-west line and are 5 miles apart. There is a fire with a bearing of N 27° E from tower 1 and N 32° W from tower 2. How far is the fire from tower 1?
The angle located at the fire is 180° − 63° − 58° = 59°.

$$\frac{x}{\sin 58^\circ} = \frac{5}{\sin 59^\circ}$$

$$x = \frac{5 \sin 58^\circ}{\sin 59^\circ} \approx 4.9 \text{ miles.}$$

- Assign the *Writing About Mathematics* on page 435 of the text.