

Chapter 6 Additional Topics in Trigonometry

Course/Section Lesson Number Date

Section 6.2 Law of Cosines

Section Objectives: Students will know how to use the Law of Cosines to solve oblique triangles and how to use Heron's Formula to find the area of a triangle.

I. Introduction (pp. 439–440)

Pace: 15 minutes

- State that for SAS and SSS, we use the **Law of Cosines**,

$$a^2 = b^2 + c^2 - 2ab \cos A \text{ or } \cos A = \frac{b^2 + c^2 - a^2}{2bc}.$$

- A proof of the Law of Cosines can be found on page 490 of the text.

Example 1. Given $a = 5$, $b = 8$, and $c = 7$, find all three angles of the triangle.

$$A = \cos^{-1} \frac{8^2 + 7^2 - 5^2}{2(8)(7)} \approx 38.2^\circ$$

$$C = \cos^{-1} \frac{5^2 + 8^2 - 7^2}{2(5)(8)} = 60^\circ$$

$$B \approx 180 - 38.2^\circ - 60^\circ = 81.8^\circ$$

Example 2. Given $C = 111^\circ$, $a = 27$, and $b = 18$, find the remaining side and two angles of the triangle.

$$c^2 = 27^2 + 18^2 - 2(27)(18)\cos 111^\circ \approx 1404.33$$

$$c \approx 37.43$$

$$\frac{\sin A}{27} \approx \frac{\sin 111^\circ}{37.43}$$

$$A \approx \sin^{-1} \frac{27 \sin 111^\circ}{37.43} \approx 42.3^\circ$$

$$B \approx 180^\circ - 111^\circ - 42.3^\circ = 26.7^\circ$$

II. Applications (p. 441)

Pace: 5 minutes

Example 3. A port is 50 miles due north of a lighthouse. A ship is 30 miles from the lighthouse at a bearing of N 37° E. How far is the ship from the port?

$$x^2 = 50^2 + 30^2 - 2(50)(30)\cos 37^\circ \approx 1004.09$$

$$x \approx 31.69 \text{ miles}$$

III. Heron's Area Formula (p. 442)

Pace: 5 minutes

- State that from the Law of Cosines comes **Heron's Area Formula**, which is

$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$$

where $s = (a + b + c)/2$.

- Discuss the *Historical Note* on page 442 of the text.

Example 4. Find the area of the triangular region with sides 50 feet, 58 feet, and 69 feet.

$$s = (50 + 58 + 69)/2 = 88.5$$

$$\text{Area} = \sqrt{88.5(88.5 - 50)(88.5 - 58)(88.5 - 69)} \approx 1423.54 \text{ ft}^2$$

- Assign the *Writing About Mathematics* on page 442 of the text.