Chapter 7 Systems of Equations and Inequalities

Section 7.1 Linear and Nonlinear Systems of Equations

Course/Section
Lesson Number
Date

Section Objectives: Students will know how to solve systems of equations by substitution and by graphing.

I. The Method of Substitution (pp. 496−498) Pace: 10 minutes

- State that we now are going to solve two or more equations with the same variables simultaneously. These are called **systems of equations.** A **solution** of a system of equations must be a solution of every equation in the system.
- State that our first method of solving a system of equations is the **method of substitution.** This method involves solving one of the equations for one of the variables, substituting for that variable in the other equation, and then solving this new equation.

Tip: Students will need to be repeatedly reminded to back-substitute to find the value of the other variable.

Example 1. Solve the following system of equations.

a) $\begin{cases} x + y = 5 \\ x - y = 3 \Rightarrow x = y + 3 \end{cases}$

$(y+3) + y = 5$

$2y = 2$

$y = 1$

$x = 1 + 3 = 4$

The solution is (4, 1).

II. Nonlinear Systems of Equations (p. 499) Pace: 5 minutes

- State that the system in Example 1 above was a linear system. The substitution method can also be used to solve systems in which one or both equations are nonlinear.

Example 2. Solve the following systems of equations.

a) $\begin{cases} x^2 + y^2 = 5 \\ x + y = 1 \Rightarrow x = 1 - y \end{cases}$

$(1-y)^2 + y^2 = 5$

$2y^2 - 2y + 1 = 5$

$y^2 - y - 2 = 0$

$(y+1)(y-2) = 0$

$y + 1 = 0 \Rightarrow y = -1 \Rightarrow x = 2$

$y - 2 = 0 \Rightarrow y = 2 \Rightarrow x = -1$

The solutions are (2, −1) and (−1, 2).

b) $\begin{cases} x^2 - y = -6 \\ x - 4y = 8 \Rightarrow x = 4y + 8 \end{cases}$

$$(4y+8)^2 - y = -6$$

$$16y^2 + 63y + 70 = 0$$

$$y = \frac{-63 \pm \sqrt{63^2 - 4(16)(70)}}{2(16)}$$

$$= \frac{-63 \pm \sqrt{-511}}{32}$$

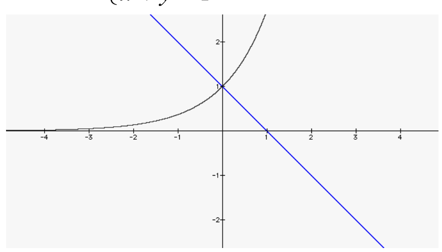
No solution.

III. Graphical Approach to Finding Solutions (p. 500) Pace: 5 minutes

- Recall that the graph of an equation in two variables is the set of all points in the rectangular coordinate system that are solutions of the equation. Therefore, if we graph the two equations of a system of equations, their points of intersection must be the points whose ordered pairs are solutions of both equations--i.e., the solutions of the system of equations.
- Discuss the *Technology* on page 500 of the text.

Example 3. Solve the system of equations.

$$\begin{cases} y = e^x \\ x + y = 1 \end{cases}$$



The solution is (0, 1).

IV. Applications (pp. 501−502) Pace: 5 minutes

- State that the **break-even point** is the point at which revenue equals cost.

Example 4. A company has fixed monthly manufacturing costs of \$12,000, and it costs \$0.95 to produce each unit of product. The company then sells each unit for \$1.25. How many units must be sold before this company breaks even?

C = 12,000 + 0.95x and R = 1.25x. For the break-even point, $C = R$..

$$\begin{cases} C = 12000 + 0.95x \\ C = 1.25x \end{cases}$$

$$1.25x = 12000 + 0.95x$$

$$0.3x = 12000$$

$$x = 40{,}000$$

- Assign the *Writing About Mathematics* on page 502 of the text.