Chapter 7 Systems of Equations and Inequalities

Course/Section
Lesson Number
Date

Section 7.2 Two-Variable Linear Systems

Section Objectives: Students will know how to solve systems of equations by elimination, and how to graphically interpret the number of solutions of a system of equations.

I. The Method of Elimination (pp. 507−509) Pace: 20 minutes

- State that we now study our third method for solving systems of equations. The **elimination method** is based on two key steps. First, obtain opposite coefficients on one of the variables. Second, add the two equations, thus eliminating this variable.

Tip: Students should be told that this method does not work for most nonlinear systems of equations. It is best to use the substitution method for nonlinear systems of equations.

Example 1. Solve the following systems of equations.

a) $\begin{cases} 2x - 3y = 7 \\ 5x + 3y = 0 \end{cases}$

$$\begin{array}{r} 2x - 3y = 7 \\ \underline{5x + 3y = 0} \\ 7x \quad\quad = 7 \\ x = 1 \\ 2(1) + 3y = 7 \\ 3y = 5 \\ y = 5/3 \end{array}$$

The solution is (1, 5/3).

b) $\begin{cases} 3x + 4y = 11 \\ x + 2y = 5 \end{cases}$

$$\begin{array}{r} 3x + 4y = 11 \\ \underline{-3x - 6y = -15} \\ -2y = -4 \\ y = 2 \\ x + 2(2) = 5 \\ x = 1 \end{array}$$

The solution is (1, 2).

c) $\begin{cases} 2x - 3y = -15 \\ 5x + 2y = 10 \end{cases} \Rightarrow \begin{cases} 4x - 6y = -30 \\ \underline{15x + 6y = 30} \end{cases}$

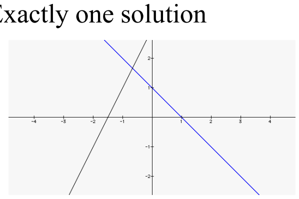
$$19x \quad = 0$$

So, $x = 0$ and $y = 5$. The solution is (0, 5).

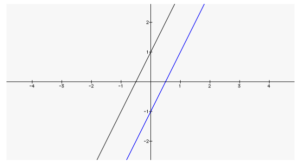
II. Graphical Interpretation of Solutions (pp. 510−512) Pace: 15 minutes

- Discuss the number of solutions a system of equation can have: one, infinitely many, or none. Then discuss the graphical interpretation that corresponds to each case. Here are the three graphs.

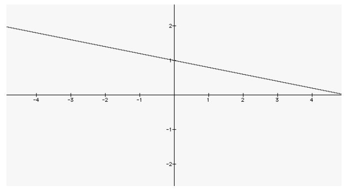
1. Exactly one solution



2. No solution



3. Infinitely many solutions (the lines are identical)



- State that in the first graph, we get exactly one solution. In the second, we get no solution. In the third, we get infinitely many solutions.
- State that if a system of linear equations has at least one solution, then it is called **consistent.** If it has no solution, then it is called **inconsistent.**

Example 2. Solve the following system of equations.

a) $\begin{cases} x+3y=5 \\ -2x-6y=1 \end{cases} \Rightarrow \begin{cases} 2x+6y=10 \\ \underline{-2x-6y=1} \end{cases}$

$$0=11$$

False. Therefore, the system has no solution.

b) $\begin{cases} 0.25x-0.5y=1 \\ -x+2y=-4 \end{cases} \Rightarrow \begin{cases} x-2y=4 \\ \underline{-x+2y=-4} \end{cases}$

$$0=0$$

Therefore, there are infinitely many solutions. In fact, the solution set is the set of all (x, y) such that $-x + 2y = -4$.

Tip: The above solution set can be written as $\{(x, y) | -x + 2y = -4\}$.

III. Applications (pp. 513−514) Pace: 10 minutes

Example 3. A man in a boat can row 8 miles downstream in 1 hour. He can row 6 miles upstream in 3 hours. How fast can the man row in still water, and what is the rate of the current?

$\begin{cases} r+c=8 \\ 3(r-c)=6 \end{cases} \Rightarrow \begin{cases} 3r+3c=24 \\ \underline{3r-3c=6} \end{cases}$

$$6r \quad = 30 \Rightarrow r=5 \Rightarrow c=3$$

The man can row 5 mph in still water, and the rate of the current is 3 mph.

Example 4. You have \$10,000 to invest in two simple interest funds. One pays 8% and the other 6%. How much should you invest in each account so that the total annual interest is \$720?

$\begin{cases} x+ \quad y=10{,}000 \\ .08x+.06y=720 \end{cases} \Rightarrow \begin{cases} -8x-8y=-80{,}000 \\ \underline{8x+6y=72{,}000} \end{cases}$

$$-2y=-8{,}000 \Rightarrow y=4{,}000 \Rightarrow x=6{,}000$$

You should invest \$6,000 at 8% and \$4,000 at 6%.