Chapter 7 Systems of Equations and Inequalities

Course/Section Lesson Number Date

Section 7.3 Multivariable Linear Systems

Section Objectives: Students will know how to use back-substitution to solve linear systems in row-echelon form, how to use Gaussian elimination to solve systems of equations, and how to solve nonsquare systems of equations.

I. Row-Echelon Form and Back-Substitution (p. 519) Pace: 5 minutes

- State that we are now going to apply the concept of elimination to systems of three linear equations in three variables. Our goal is to transform the system of equations into a system of equations to which back-substitution can be applied. We will call this form **row-echelon form.**

Example 1. Solve the system of equations, which is in row-echelon form.

$$\begin{cases} x - y + 2z = 1 \\ \quad y + z = 3 \\ \quad\quad z = 2 \end{cases}$$

From the third equation, we see that $z = 2$. Substitute this value into the second equation and solve to get $y = 1$. Substitute both of these values into the first equation and solve to get $x = -2$. The solution is the **ordered triple** $(-2, 1, 2)$.

II. Gaussian Elimination (pp. 520−523) Pace: 20 minutes

- State that two systems of equations are *equivalent* if they have the same solution set.
- State the following **row operations** that will transform a system of equations into an equivalent system of equations.
  1. Interchange two equations.
  2. Multiply any of the equations by a nonzero constant.
  3. Add a multiple of one equation in the system to another equation to replace the latter equation.
- State that solving a system of equations by transforming it into row-echelon form is called **Gaussian elimination.**

Example 2. Solve the following systems of equations by Gaussian elimination.

a) $\begin{cases} x - 2y = 1 \\ 2x - 3y = 6 \end{cases} \Rightarrow \begin{cases} x - 2y = 1 \\ \quad y = 4 \end{cases} \Rightarrow (9, 4)$

b)

$$\begin{cases} x - 3y + 2z = 1 \\ 2x - 5y + z = -5 \\ 3x + y - 2z = -1 \end{cases} \Rightarrow \begin{cases} x - 3y + 2z = 1 \\ y - 3z = -7 \\ 10y - 8z = -4 \end{cases} \Rightarrow$$

$$\begin{cases} x - 3y + 2z = 1 \\ y - 3z = -7 \\ 22z = 66 \end{cases} \Rightarrow \begin{cases} x - 3y + 2z = 1 \\ y - 3z = -7 \\ z = 3 \end{cases} \Rightarrow (1, 2, 3)$$

c) $\begin{cases} x + 3y - 2z = 1 \\ -x - 2y + 6z = -2 \\ 3x + 11y + 2z = 6 \end{cases} \Rightarrow \begin{cases} x + 3y - 2z = 1 \\ y + 4z = -1 \\ 2y + 8z = 3 \end{cases} \Rightarrow \begin{cases} x + 3y - 2z = 1 \\ y + 4z = -1 \\ 0 = 5 \end{cases}$

As we saw in the previous section, the last equation indicates that there are no solutions.

- Discuss the different ways that three planes can intersect or not intersect; see Figures 7.12 – 7.16 on page 522 of the text.

Example 3. Solve the following system of equations by Gaussian elimination.

$$\begin{cases} x + y + z = 2 \\ 2x + 3y + 4z = 5 \\ x - z = 1 \end{cases} \Rightarrow \begin{cases} x + y + z = 2 \\ y + 2z = 1 \\ -y - 2z = -1 \end{cases} \Rightarrow \begin{cases} x + y + z = 2 \\ y + 2z = 1 \\ 0 = 0 \end{cases}$$

Letting $z = a$ and back-substituting, we get $(a + 1, 1 - 2a, a)$, a is a real number.

III. Nonsquare Systems (p. 524) Pace: 5 minutes

- State that with a **nonsquare** system of equations (the number of equations does not equal the number of variables), we will never have exactly one solution.

Example 4. Solve the following system of equations by Gaussian elimination.

$$\begin{cases} x - 4y + 2z = 5 \\ 3x + 5y - 2z = 1 \end{cases} \Rightarrow \begin{cases} x - 4y + 2z = 5 \\ 17y - 8z = -14 \end{cases} \Rightarrow \begin{cases} x - 4y + 2z = 5 \\ y - \frac{8}{17}z = -\frac{14}{17} \end{cases}$$

The solution is $\left(\frac{29}{17} - \frac{2}{17}a, \frac{8}{17}a - \frac{14}{17}, a\right)$.

IV. Applications (pp. 525−526) Pace: 10 minutes

Example 5. Find the equation of the parabola $y = ax^2 + bx + c$ that passes through the points (1, 6), (−1, 4), and (2, 13).

$$\left.\begin{aligned} (1,6) &\Rightarrow a + b + c = 6 \\ (-1,4) &\Rightarrow a - b + c = 4 \\ (2,13) &\Rightarrow 4a + 2b + c = 13 \end{aligned}\right\} \Rightarrow \begin{cases} a + b + c = 6 \\ b = 1 \\ c = 3 \end{cases}$$

$y = 2x^2 + x + 3$