Chapter 7 Systems of Equations and Inequalities

Course/Section
Lesson Number
Date

Section 7.6 Linear Programming

Section Objectives: Students will know how to solve linear programming problems.

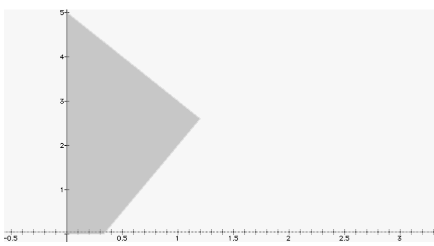
I. Linear Programming: A Graphical Approach (pp. 552−555)
Pace: 20 minutes

- State that **linear programming** involves optimizing an objective function subject to constraints. In our studies in this section, the **objective function** will be linear and the **constraints** will be a system of linear inequalities.
- State that if a linear programming problem has a solution, it occurs at a vertex of the constraint region. Hence to solve a linear programming problem, we will find the coordinates of the vertices of the constraint region and evaluate the objective function at these points.

Example 1. Find the maximum value of $z = 4x + y$, subject to the following constraints.

$$\left.\begin{aligned} 2x + y &\le 5 \\ 3x - y &\le 1 \\ x &\ge 0 \\ y &\ge 0 \end{aligned}\right\}$$

The constraints form the graph below. From the graph, we can see that the vertices are at (0, 0), (0, 5), (1/3, 0) and (6/5, 23/5).



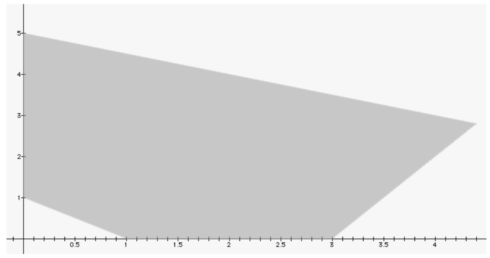
(*x*, *y*)	(0, 0)	(0, 5)	(1/3, 0)	(6/5, 23/5)
z	0	5	4/3	47/5

So, the maximum value of *z* is 47/5, and it occurs at (6/5, 23/5).

- Mention the *Historical Note* on page 554 of the text.

Example 2. Find the minimum value of $z = 2x + y$, subject to the following constraints.

$$\left.\begin{aligned} x + y &\ge 1 \\ x + 2y &\le 10 \\ 2x - y &\le 6 \\ x &\ge 0 \\ y &\ge 0 \end{aligned}\right\}$$

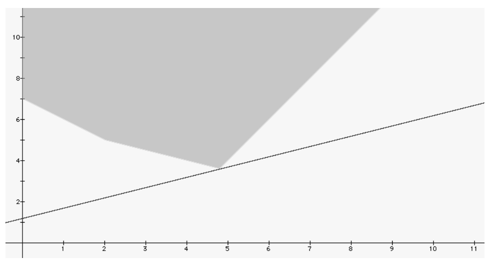


The minimum is $z = 1$ at (0, 1)

Tip: If you pick a value for the objective function and graph the objective line, then you may be able to see at which vertex the maximum or minimum occurs. Therefore, we only need to find the one vertex.

Example 3. Find the minimum value of $z = -x + 2y$, subject to the following constraints.

$$\left.\begin{aligned} x + y &\geq 7 \\ x + 2y &\geq 12 \\ 2x - y &\leq 6 \\ x &\geq 0 \end{aligned}\right\}$$



From the graph it is clear which vertex we need. The minimum value of $z = 2.4$ occurs at (4.8, 3.6).

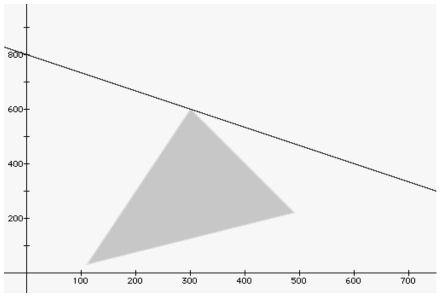
II. Applications (pp. 556−557) Pace: 15 minutes

Example 4. A company earns $2 profit on each unit of Brand X and $3 profit on each unit of Brand Y. Because of union requirements, the total of twice the number of units of Brand X and the number of units of Brand Y cannot exceed 1200 units. Also, the difference in the number of units of Brand X and twice the number of units of Brand Y cannot exceed 50. Company research shows that the difference in three times the number of units of Brand X and the number of units of Brand Y must be at least 300. How many units of each should this company produce to maximize its profit?

Objective Function: $P = 2x + 3y$

Constraints:

$$\begin{cases} 2x + y \leq 1200 \\ x - 2y \leq 50 \\ 3x - y \geq 300 \\ x \geq 0 \\ y \geq 0 \end{cases}$$

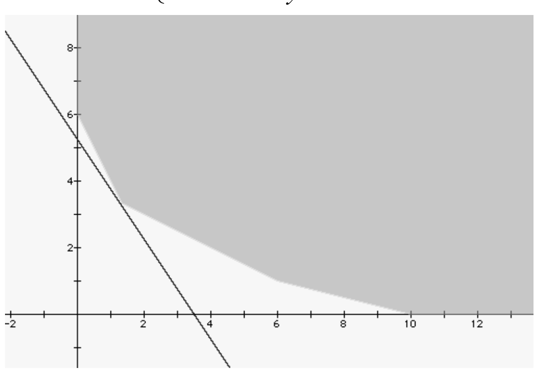


The maximum profit occurs at (300, 600).
The company should produce 300 units of Brand X and 600 units of Brand Y.

Example 5. The minimum daily requirements from a powdered dietary supplement are 600 units of vitamin A, 800 units of vitamin B, and 1000 units of vitamin C. A pound of dietary powder X has 200 units of vitamin A, 100 units of vitamin B, and 100 units of vitamin C and costs \$3 per pound. A pound of dietary powder Y has 100 units of vitamin A, 200 units of vitamin B, and 400 units of vitamin C and costs \$2 per pound. How many pounds of each powder should be consumed each day to minimize the cost and still meet the minimum daily requirements?

Objective Function: $C = 3x + 2y$

Constraints:
$$\begin{cases} 200x + 100y \geq 600 \\ 100x + 200y \geq 800 \\ 100x + 400y \geq 1000 \\ x \geq 0 \\ y \geq 0 \end{cases}$$



The minimum cost occurs at (4/3, 10/3).
The minimum cost is \$10.67 per day, and this occurs when 4/3 pounds of powder X and 10/3 pounds of powder Y are consumed each day.