Chapter 8 Matrices and Determinants

Section 8.2 Operations with Matrices

Course/Section
Lesson Number
Date

Section Objectives: Students will know how to add and subtract matrices, multiply matrices by real numbers, and multiply two matrices.

I. Equality of Matrices. (p. 587) Pace: 5 minutes

- State that matrices will be represented by uppercase letters and also by $A = [a_{ij}]$. Furthermore, two $m \times n$ matrices $A = [a_{ij}]$ and $B = [b_{ij}]$ are equal if and only if $a_{ij} = b_{ij}$ for all $i = 1, 2, \ldots, m$ and $j = 1, 2, \ldots, n$.

II. Matrix Addition and Scalar Multiplication (pp. 588−591)
Pace: 10 minutes

- Discuss the *Historical Note* on page 588 of the text.
- State the definition of **matrix addition.**
  If $A = [a_{ij}]$ and $B = [b_{ij}]$ are two $m \times n$ matrices, then their sum is the $m \times n$ matrix given by

  $$A + B = [a_{ij} + b_{ij}].$$

  Note that the matrices must be of the same order for addition to take place.

  Example 1. Find the following sums.

  a) $\begin{bmatrix} 2 & -5 \\ 4 & 3 \end{bmatrix} + \begin{bmatrix} 8 & -9 \\ 1 & -3 \end{bmatrix} = \begin{bmatrix} 10 & -14 \\ 5 & 0 \end{bmatrix}$

  b) $\begin{bmatrix} 4 & -5 \\ -6 & 2 \\ 3 & 8 \end{bmatrix} + \begin{bmatrix} 8 & 9 \\ 6 & -1 \\ 3 & -2 \end{bmatrix} = \begin{bmatrix} 12 & 4 \\ 0 & 1 \\ 6 & 6 \end{bmatrix}$

- State the definition of **scalar multiplication.**
  If $A = [a_{ij}]$ is an $m \times n$ matrix and c is a scalar, then the product of A and c is the $m \times n$ matrix given by

  $$cA = [ca_{ij}].$$

- State that the symbol $-A$ represents the additive inverse of A and equals $(-1)A$. Moreover, $A - B = A + (-B)$.

Example 2. For the following matrices, find (a) $2A$ and (b) $A - B$.

$$A = \begin{bmatrix} 4 & -2 & 3 \\ 1 & 8 & -6 \\ 5 & -7 & 1 \end{bmatrix} \text{ and } B = \begin{bmatrix} -1 & 6 & 2 \\ 4 & -3 & 5 \\ 9 & -2 & 3 \end{bmatrix}$$

a) $$2A = 2\begin{bmatrix} 4 & -2 & 3 \\ 1 & 8 & -6 \\ 5 & -7 & 1 \end{bmatrix} = \begin{bmatrix} 8 & -4 & 6 \\ 2 & 16 & -12 \\ 10 & -14 & 2 \end{bmatrix}$$

b) $$A - B = \begin{bmatrix} 4 & -2 & 3 \\ 1 & 8 & -6 \\ 5 & -7 & 1 \end{bmatrix} - \begin{bmatrix} -1 & 6 & 2 \\ 4 & -3 & 5 \\ 9 & -2 & 3 \end{bmatrix} = \begin{bmatrix} 5 & -8 & 1 \\ -3 & 11 & -11 \\ -4 & -5 & -2 \end{bmatrix}$$

- Define the zero matrix to be the $m \times n$ matrix given by $O = [0]$.
- State the following properties of matrix addition and scalar multiplication. Let A, B, and C be $m \times n$ matrices and let c and d be scalars.
  1. $A + O = O + A = A$
  2. $A + B = B + A$
  3. $A + (B + C) = (A + B) + C$
  4. $(cd)A = c(dA)$
  5. $1A = A$
  6. $c(A + B) = cA + cB$
  7. $(c + d)A = cA + dA$

- Discuss the *Technology* on page 590 of the text.

Example 3. Solve the matrix equation $A - 2X = B$, where

$$A = \begin{bmatrix} 2 & -1 \\ 4 & 3 \end{bmatrix} \text{ and } B = \begin{bmatrix} -3 & -6 \\ 5 & 1 \end{bmatrix}.$$

$X = -0.5(B - A)$

$$X = -\frac{1}{2}\left(\begin{bmatrix} -3 & -6 \\ 5 & 1 \end{bmatrix} - \begin{bmatrix} 2 & -1 \\ 4 & 3 \end{bmatrix}\right)$$

$$= -\frac{1}{2}\begin{bmatrix} -5 & -5 \\ 1 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{5}{2} & \frac{5}{2} \\ -\frac{1}{2} & 1 \end{bmatrix}$$

III. Matrix Multiplication (pp. 592−594) Pace: 15 minutes

- State the definition of **matrix multiplication**.
  If $A = [a_{ij}]$ is an $m \times n$ matrix and $B = [b_{ij}]$ is an $n \times p$ matrix, the product AB is an $m \times p$ matrix given by
  $AB = [c_{ij}]$
  where $c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + \cdots + a_{in}b_{nj}$.

Tip: Describe this process as taking the elements in the ith row of A, multiplying them by the corresponding elements in the jth column of B, and then summing these products.

Example 4. Find the following products.

a) $\begin{bmatrix} -1 & 0 \\ 6 & 2 \end{bmatrix}\begin{bmatrix} 5 & -2 & 7 \\ 1 & 0 & 3 \end{bmatrix} = \begin{bmatrix} -5 & 2 & -7 \\ 32 & -12 & 48 \end{bmatrix}$

b) $\begin{bmatrix} 1 & 1 & -2 \\ 0 & 6 & 3 \\ 2 & -2 & -3 \end{bmatrix}\begin{bmatrix} -2 & 0 & 5 \\ 4 & 3 & -2 \\ -1 & -4 & 2 \end{bmatrix} = \begin{bmatrix} 4 & 11 & -1 \\ 21 & 6 & -6 \\ -9 & 6 & 8 \end{bmatrix}$

c) $\begin{bmatrix} 2 & -5 \\ 9 & 7 \end{bmatrix}\begin{bmatrix} 3 & -1 \\ 3 & 6 \end{bmatrix} = \begin{bmatrix} -9 & -32 \\ 48 & 33 \end{bmatrix}$

d) $\begin{bmatrix} 3 & -1 \\ 3 & 6 \end{bmatrix}\begin{bmatrix} 2 & -5 \\ 9 & 7 \end{bmatrix} = \begin{bmatrix} -3 & -22 \\ 60 & 27 \end{bmatrix}$

- Ask the class what the last two examples suggest. For most matrices, $AB \neq BA$.
- Define I_n to be the $n \times n$ matrix for which every element along the main diagonal is 1 and all other elements are 0.
- State the following properties of matrix multiplication.
  Let A, B, and C be matrices and let c be a scalar.

1. $AI_n = I_nA = A$
2. $A(BC) = (AB)C$
3. $A(B + C) = AB + AC$
4. $(A + B)C = AC + BC$
5. $c(AB) = (cA)B = A(cB)$

IV. Applications (pp. 595−596) Pace: 5 minutes

Example 5. Write the following system of equations as a matrix equation.

$$\begin{cases} 2x + 3y + 4z = 5 \\ 3x + 9y - 5z = 0 \\ 5x - 3y + 5z = 9 \end{cases} \Rightarrow \begin{bmatrix} 2 & 3 & 4 \\ 3 & 9 & -5 \\ 5 & -3 & 5 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ 0 \\ 9 \end{bmatrix}$$

Example 6. Two tennis teams submit equipment requests to their sponsors.

	Women's Team	*Men's Team*
Balls	50	48
Rackets	12	15
Shoes	15	18

Each can of balls costs $5, each racket costs $129, and each pair of shoes costs $79. Use matrix multiplication to find the total cost of equipment for each team.

$$\begin{bmatrix} 5 & 129 & 79 \end{bmatrix}\begin{bmatrix} 50 & 48 \\ 12 & 15 \\ 15 & 18 \end{bmatrix} = \begin{bmatrix} 2983 & 3597 \end{bmatrix}$$

So, the total cost of equipment for the women's team is $2,983 and the total cost of equipment for the men's team is $3,597.