

# Chapter 8 Matrices and Determinants

Course/Section Lesson Number Date
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## Section 8.3 The Inverse of a Square Matrix

**Section Objectives:** Students will know how to find the inverses of matrices and use inverse matrices to solve systems of linear equations.

### I. The Inverse of a Matrix (p. 602)

Pace: 5 minutes

- State that we want to solve systems of equations by expressing them as matrix equations, and then solving as if we were solving  $ax = b$ --i.e., multiplying both sides by the inverse of  $a$ .
- Define the inverse of an  $n \times n$  matrix  $A$ , if it exists, to be the  $n \times n$  matrix  $A^{-1}$  such that  $AA^{-1} = A^{-1}A = I_n$ .

**Example 1.** Show that  $B$  is the inverse of  $A$ , where

$$A = \begin{bmatrix} -5 & 6 \\ -1 & 1 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 & -6 \\ 1 & -5 \end{bmatrix}.$$
$$AB = \begin{bmatrix} -5 & 6 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & -6 \\ 1 & -5 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$
$$BA = \begin{bmatrix} 1 & -6 \\ 1 & -5 \end{bmatrix} \begin{bmatrix} -5 & 6 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

### II. Finding Inverse Matrices (pp. 603–605)

Pace: 15 minutes

- To dispel any thoughts of magic, show students that the process we will state shortly comes from the following.

**Example 2.** Find the inverse of  $A = \begin{bmatrix} 4 & 5 \\ 7 & 9 \end{bmatrix}$ .

We need to find the matrix  $B = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$  such that  $AB = I_2$ . By

multiplying, we see that we need to solve the following two systems of equations.

$$\begin{cases} 4a + 5c = 1 \\ 7a + 9c = 0 \end{cases} \Rightarrow \begin{bmatrix} 4 & 5 & 1 \\ 7 & 9 & 0 \end{bmatrix}$$
$$\begin{cases} 4b + 5d = 0 \\ 7b + 9d = 1 \end{cases} \Rightarrow \begin{bmatrix} 4 & 5 & 0 \\ 7 & 9 & 1 \end{bmatrix}$$
$$\Rightarrow \left[ \begin{array}{cc|cc} 4 & 5 & 1 & 0 \\ 7 & 9 & 0 & 1 \end{array} \right] \Rightarrow \left[ \begin{array}{cc|cc} 1 & 0 & 9 & -5 \\ 0 & 1 & -7 & 4 \end{array} \right] \Rightarrow$$
$$B = \begin{bmatrix} 9 & -5 \\ -7 & 4 \end{bmatrix}$$

- State the following **steps for finding an inverse matrix**.  
Let  $A$  be a square matrix of order  $n$ .
  1. Form the  $n \times 2n$  matrix  $[A:I_n]$ .
  2. Transform this matrix into reduced row-echelon form.
  3. If this new matrix is of the form  $[I_n:B]$ , then  $A$  is invertible and  $B = A^{-1}$ .

**Example 3.** Find the inverse of

$$A = \begin{bmatrix} 1 & 2 & 5 \\ 3 & 7 & 14 \\ -1 & 4 & -3 \end{bmatrix}$$

$$\left[ \begin{array}{ccc|ccc} 1 & 2 & 5 & 1 & 0 & 0 \\ 3 & 7 & 14 & 0 & 1 & 0 \\ -1 & 4 & -3 & 0 & 0 & 1 \end{array} \right] \Rightarrow \left[ \begin{array}{ccc|ccc} 1 & 2 & 5 & 1 & 0 & 0 \\ 0 & 1 & -1 & -3 & 1 & 0 \\ 0 & 6 & 2 & 1 & 0 & 1 \end{array} \right] \Rightarrow$$

$$\left[ \begin{array}{ccc|ccc} 1 & 0 & 7 & 7 & -2 & 0 \\ 0 & 1 & -1 & -3 & 1 & 0 \\ 0 & 0 & 8 & 19 & -6 & 1 \end{array} \right] \Rightarrow \left[ \begin{array}{ccc|ccc} 1 & 0 & 7 & 7 & -2 & 0 \\ 0 & 1 & -1 & -3 & 1 & 0 \\ 0 & 0 & 1 & 19/8 & -3/4 & 1/8 \end{array} \right] \Rightarrow$$

$$\left[ \begin{array}{ccc|ccc} 1 & 0 & 7 & -77/8 & 13/4 & -7/8 \\ 0 & 1 & -1 & -5/8 & 1/4 & 1/8 \\ 0 & 0 & 1 & 19/8 & -3/4 & 1/8 \end{array} \right]$$

$$A^{-1} = \frac{1}{8} \begin{bmatrix} -77 & 26 & -7 \\ -5 & 2 & 1 \\ 19 & -6 & 1 \end{bmatrix}$$

**III. The Inverse of a  $2 \times 2$  Matrix** (p. 606)

Pace: 5 minutes

- State the following **formula for finding the inverse of a  $2 \times 2$  matrix**.

If  $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$  and  $ad - bc \neq 0$ , then

$$A^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

**Example 4.** Find the inverse of

$$A = \begin{bmatrix} 3 & 2 \\ 9 & 5 \end{bmatrix}$$

$$A^{-1} = -\frac{1}{3} \begin{bmatrix} 5 & -2 \\ -9 & 3 \end{bmatrix}$$

**IV. Systems of Linear Equations** (p. 607)

Pace: 5 minutes

- State that if  $A$  is invertible, then the system of equations represented by  $AX = B$  has a unique solution  $X = A^{-1}B$ .

**Example 5.** Solve the following system of equations.

$$\begin{cases} 2x - 5y = 8 \\ 3x - 8y = 1 \end{cases}$$

$$\begin{bmatrix} 2 & -5 \\ 3 & -8 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 8 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 & -5 \\ 3 & -8 \end{bmatrix}^{-1} \begin{bmatrix} 8 \\ 1 \end{bmatrix} = \begin{bmatrix} 8 & -5 \\ 3 & -2 \end{bmatrix} \begin{bmatrix} 8 \\ 1 \end{bmatrix} = \begin{bmatrix} 59 \\ 22 \end{bmatrix}$$

- Discuss the *Technology* on page 607 of the text.