Chapter 8 Matrices and Determinants

Course/Section
Lesson Number
Date

Section 8.4 The Determinant of a Square Matrix

Section Objectives: Students will know how to find minors, cofactors, and determinants of square matrices.

I. The Determinant of a 2 × 2 Matrix (pp. 611−612) Pace: 5 minutes

- State that to each square matrix there corresponds a unique real number called the **determinant** of the matrix.
- State that if $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, then the determinant of A is

$$\det(A) = |A| = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc.$$

Example 1. Find the determinant of

$$A = \begin{bmatrix} -6 & 2 \\ 7 & 1 \end{bmatrix}$$

$$\begin{vmatrix} -6 & 2 \\ 7 & 1 \end{vmatrix} = -6 \cdot 1 - 2 \cdot 7 = -20$$

II. Minors and Cofactors (p. 613) Pace: 10 minutes

- State that to find the determinant of a matrix larger than order 2, we need to use minors and cofactors. State the following definitions.
  If A is a square matrix, the **minor** of a_{ij}, denoted by M_{ij}, is the determinant of the square matrix of order $n - 1$ formed by deleting the ith row and the jth column of A. The **cofactor** of a_{ij}, denoted by C_{ij}, is
  $C_{ij} = (-1)^{i+j} M_{ij}$.

Example 2. Find M_{11}, C_{11}, M_{21}, C_{21}, M_{33}, and C_{33}.

$$A = \begin{bmatrix} 5 & 6 & 2 \\ -1 & -8 & 3 \\ 7 & -2 & 9 \end{bmatrix}$$

$$M_{11} = \begin{vmatrix} -8 & 3 \\ -2 & 9 \end{vmatrix} = -66 \Rightarrow C_{11} = (-1)^{1+1}(-66) = -66$$

$$M_{21} = \begin{vmatrix} 6 & 2 \\ -2 & 9 \end{vmatrix} = 58 \Rightarrow C_{21} = (-1)^{2+1}(58) = -58$$

$$M_{33} = \begin{vmatrix} 5 & 6 \\ -1 & -8 \end{vmatrix} = -34 \Rightarrow C_{33} = (-1)^{3+3}(-34) = -34$$

- Mention that the $(-1)^{i+j}$ factor of the cofactor alternates in sign.

III. The Determinant of a Square Matrix (pp. 614−615) Pace: 10 minutes

- State that if A is a square matrix, then the determinant of A is the sum of the products of each element in any row (or column) of A and their cofactors.

Example 3. Find the determinants of the following matrices.

a)

$$A = \begin{bmatrix} 2 & -1 & 5 \\ 8 & -3 & 2 \\ 1 & -2 & 3 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 2 & -1 & 5 \\ 8 & -3 & 2 \\ 1 & -2 & 3 \end{vmatrix} = 2\begin{vmatrix} -3 & 2 \\ -2 & 3 \end{vmatrix} - (-1)\begin{vmatrix} 8 & 2 \\ 1 & 3 \end{vmatrix} + 5\begin{vmatrix} 8 & -3 \\ 1 & -2 \end{vmatrix}$$

$$= 2(-5) + (22) + 5(-13) = -53$$

b)

$$A = \begin{bmatrix} 1 & 0 & 3 & 5 \\ 2 & 0 & -5 & 4 \\ -1 & 1 & 9 & -3 \\ -4 & 0 & -5 & 2 \end{bmatrix}$$

$$|A| = -1\begin{vmatrix} 1 & 3 & 5 \\ 2 & -5 & 4 \\ -4 & -5 & 2 \end{vmatrix} = -1\left(1\begin{vmatrix} -5 & 4 \\ -5 & 2 \end{vmatrix} - 3\begin{vmatrix} 2 & 4 \\ -4 & 2 \end{vmatrix} + 5\begin{vmatrix} 2 & -5 \\ -4 & -5 \end{vmatrix}\right)$$

$$= -1(10 - 3(20) + 5(-30)) = 200$$