Chapter 9 Sequences, Series, and Probability

Course/Section
Lesson Number
Date

Section 9.1 Sequences and Series

Section Objectives: Students will know how to use sequence, factorial, and summation notation to write the terms and sum of a sequence, and how to find the sum of an infinite series.

I. Sequences (pp. 642−644) Pace: 10 minutes

- State that an **infinite sequence** is a function whose domain is the set of positive integers. The function values are called the **terms** of the sequence. If the domain of the function consists of the first n positive integers, then the sequence is a **finite sequence**.
- Discuss sequence notation by saying that instead of writing $f(n)$, we write a_n.

Example 1. Write the first four terms of the following sequences.

a) $a_n = 2n + 5$
$a_1 = 2(1) + 5 = 7$
$a_2 = 2(2) + 5 = 9$
$a_3 = 2(3) + 5 = 11$
$a_4 = 2(4) + 5 = 13$

b) $b_n = 3^{n-1}$
$b_1 = 3^{1-1} = 1$
$b_2 = 3^{2-1} = 3$
$b_3 = 3^{3-1} = 9$
$b_4 = 3^{4-1} = 27$

c) $c_n = \dfrac{(-1)^n}{n^2+1}$

$c_1 = \dfrac{(-1)^1}{1^2+1} = -\dfrac{1}{2}$

$c_2 = \dfrac{(-1)^2}{2^2+1} = \dfrac{1}{5}$

$c_3 = \dfrac{(-1)^3}{3^2+1} = -\dfrac{1}{10}$

$c_4 = \dfrac{(-1)^1}{4^2+1} = \dfrac{1}{17}$

- Discuss the *Technology* feature on page 643 of the text.

II. Factorial Notation (pp. 644−645) Pace: 5 minutes

- Define ***n* factorial** to be $n! = n(n-1)(n-2)\cdots 2\cdot 1$, where n is a positive integer. As a special case, $0! = 1$.
- State the difference between $2n!$ and $(2n)!$.

$2n! = 2n(n-1)(n-2)\cdots 2\cdot 1$

$(2n)! = 2n(2n-1)(2n-2)\cdots 2\cdot 1$

Example 2. Write the first four terms of the following sequence.

$$a_n = \frac{n^2}{n!}$$

$$a_1 = \frac{1^2}{1!} = 1,\ a_2 = \frac{2^2}{2!} = 2,\ a_3 = \frac{3^2}{3!} = \frac{3}{2},\ a_4 = \frac{4^2}{4!} = \frac{2}{3}$$

Example 3. Evaluate the factorial expressions.

a) $\dfrac{10!}{2!\cdot 8!} = \dfrac{10\cdot 9\cdot 8!}{2\cdot 8!} = \dfrac{10\cdot 9}{2} = 5\cdot 9 = 45$

b) $\dfrac{n!}{(n+1)!} = \dfrac{n!}{(n+1)n!} = \dfrac{1}{n+1}$

Example 4. Write the most apparent nth term of the following sequences.

a) $0, 3, 8, 15, \ldots = 1^2 - 1, 2^2 - 1, 3^2 - 1, 4^2 - 1, \ldots$

$a_n = n^2 - 1.$

b) $\frac{1}{2}, \frac{1}{6}, \frac{1}{24}, \frac{1}{120}, \ldots = \frac{1}{2!}, \frac{1}{3!}, \frac{1}{4!}, \frac{1}{5!}, \ldots$

$$b_n = \frac{1}{(n+1)!}$$

III. Summation Notation (pp. 646−647) Pace: 10 minutes

- Define **summation notation** by

$$\sum_{i=1}^{n} a_i = a_1 + a_2 + a_3 + \cdots + a_n$$

where i is called the **index,** n is the **upper limit,** and 1 is the **lower limit.**

Example 5. Find each of the following sums.

a) $\sum_{i=1}^{4} 3i + 1 = 4 + 7 + 10 + 13 = 34$

b) $\sum_{i=1}^{5} (-1)^{i-1} i! = 1 + (-2) + 6 + (-24) + 120 = 101$

- State the following **properties of sums.**

1. $\sum_{i=1}^{n} ca_i = c\sum_{i=1}^{n} a_i$, c is a constant.
2. $\sum_{i=1}^{n} (a_i \pm b_i) = \sum_{i=1}^{n} a_i \pm \sum_{i=1}^{n} b_i$

IV. Series (p. 647) Pace: 5 minutes

- State that if $a_1, a_2, a_3, \ldots$ is an infinite sequence, then we call

$$\sum_{i=1}^{n} a_i = a_1 + a_2 + a_3 + \cdots + a_n$$

the ***n*th partial sum** of the sequence. The nth partial sums of the sequence themselves form an infinite sequence. This type of infinite sequence is called an **infinite series,** and is denoted by

$$\sum_{i=1}^{\infty} a_i .$$

V. Application (p. 648) Pace: 5 minutes

Example 6. If a deposit of \$50 is made each month into an account that earns 6% interest compounded monthly, then the balance in the account after n months is $A_n = 50(200)[(1.005)^n - 1]$. Find the balance in the account after 5 years.

$n = 60 \Rightarrow A_{60} = 50(200)[(1.005)^{60} - 1] = \$3,488.50$