

Chapter 9 Sequences, Series, and Probability

Course/Section
Lesson Number
Date

Section 9.2 Arithmetic Sequences and Partial Sums

Section Objectives: Students will know how to recognize, write, and manipulate arithmetic sequences, and how to find the n th partial sum of an arithmetic sequence.

I. Arithmetic Sequences (pp. 653–655) Pace: 15 minutes

- Define an **arithmetic sequence** to be a sequence that satisfies $a_n - a_{n-1} = d$, (or $a_n = a_{n-1} + d$), where d is called the **common difference**.

Example 1. Find the common difference in the following arithmetic sequences.

a) 5, 8, 11, 14, ...
 $d = a_2 - a_1 = 8 - 5 = 3$

b) 15, 11, 7, 3, ...
 $d = a_2 - a_1 = 11 - 15 = -4$

- Develop the **n th term formula** for an arithmetic sequence by stating

$$\begin{aligned}a_1 &= a_1 \\a_2 &= a_1 + d \\a_3 &= a_2 + d = (a_1 + d) + d = a_1 + 2d \\a_4 &= a_3 + d = (a_1 + 2d) + d = a_1 + 3d \\&\vdots \\a_n &= a_1 + (n - 1)d\end{aligned}$$

Example 2. Find the n th term of the arithmetic sequence with common difference 5 and first term 9.

$$a_n = 9 + (n - 1)5 = 9 + 5n - 5 = 5n + 4$$

Example 3. Find the n th term of the arithmetic sequence with fifth term 19 and ninth term 27.

$$\left. \begin{aligned}a_5 &= a_1 + 4d = 19 \\a_9 &= a_1 + 8d = 27\end{aligned} \right\} \Rightarrow 4d = 8 \Rightarrow d = 2 \Rightarrow a_1 = 11$$

$$a_n = 11 + (n - 1)2 = 2n + 9$$

II. The Sum of a Finite Arithmetic Sequence (pp. 656–657) Pace: 10 minutes

- Develop the sum of the first n terms of an arithmetic sequence by stating

$$S_n = a_1 + a_2 + a_3 + \cdots + a_n = a_1 + (a_1 + d) + (a_1 + 2d) + \cdots + [a_1 + (n - 1)d]$$

$$S_n = a_n + a_{n-1} + a_{n-2} + \cdots + a_1 = a_n + (a_n - d) + (a_n - 2d) + \cdots + [a_n - (n - 1)d]$$

$$2S_n = (a_1 + a_n) + (a_1 + a_n) + (a_1 + a_n) + \cdots + (a_1 + a_n)$$

$$2S_n = n(a_1 + a_n)$$

$$S_n = n \frac{(a_1 + a_n)}{2}$$

Example 4. Evaluate. $\sum_{n=1}^{20} 7n + 1 = 20 \frac{(8 + 141)}{2} = 1,490$

Example 5. Find the 15th partial sum of the sequence 2, 5, 8, 11, ...

$$a_{15} = 2 + 14 \cdot 3 = 44$$

$$S_{15} = 15 \frac{(2 + 44)}{2} = 345$$

III. Applications (pp. 657–658)

Pace: 10 minutes

Example 6. Each row of an auditorium has two more seats than the preceding row. Find the seating capacity of the auditorium if the front row has 30 seats and there are 40 rows.

$$a_{40} = 30 + 39 \cdot 2 = 108$$

$$S_{40} = 40 \frac{(30 + 108)}{2} = 2,760$$

Example 7. Suppose you put \$100 under your mattress at the end of the month. You continue to put money under your mattress each month and increase the amount by \$5 each time. How much money is under your mattress after one year?

$$a_{12} = 100 + 11 \cdot 5 = 155$$

$$S_{12} = 12 \frac{(100 + 155)}{2} = \$1,530$$

- Assign the *Writing About Mathematics* on page 658 of the text.