Chapter 9 Sequences, Series, and Probability

Course/Section
Lesson Number
Date

Section 9.3 Geometric Sequences and Series

Section Objectives: Students will know how to recognize, write, and manipulate geometric sequences.

I. Geometric Sequences (pp.663−665) Pace: 15 minutes

- State that a geometric sequence is a sequence that satisfies $a_n = a_{n-1}r$ (or $a_n/a_{n-1} = r$) where r is called the **common ratio.**

Example 1. Find the common ratio of the following geometric sequences.

a) 3, 6, 12, 24, …
$r = 6/3 = 2$

b)
$1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \ldots$
$r = \frac{\frac{1}{2}}{1} = \frac{1}{2}$

- Develop the nth term of a geometric sequence by stating
$a_1 = a_1$
$a_2 = a_1r$
$a_3 = a_2r = (a_1r)r = a_1r^2$
$a_4 = a_3r = (a_1r^2)r = a_1r^3$
⋮
$a_n = a_1r^{n-1}$

Example 2. Find the nth term for the geometric sequence with first term 5 and common ratio 2.
$a_n = 5(2^{n-1})$

Example 3. Find the twentieth term of the geometric sequence 1, 3, 9, 27, …
$a_{20} = 1(3^{19}) = 1{,}162{,}261{,}467$

Example 4. Find the fifteenth term of the geometric sequence with a third term of 5/4 and a sixth term of 5/32.

$$\left.\begin{aligned} a_3 &= a_1r^2 = \frac{5}{4} \\ a_6 &= a_1r^5 = \frac{5}{32} \end{aligned}\right\} \Rightarrow \frac{5}{4}r^3 = \frac{5}{32} \Rightarrow r^3 = \frac{1}{8} \Rightarrow r = \frac{1}{2} \Rightarrow a_1 = 5$$

$$a_{15} = 5\left(\frac{1}{2}\right)^{14} = \frac{5}{16{,}384}$$

II. The Sum of a Finite Geometric Sequence (p. 666) Pace: 10 minutes

- Develop the nth partial sum of a finite geometric sequence by stating

$$S_n = a_1 + a_1r + a_1r^2 + \cdots + a_1r^{n-1}$$
$$\underline{\quad rS_n = a_1r + a_1r^2 + a_1r^3 + \cdots + a_1r^n}$$
$$S_n - rS_n = a_1 - a_1r^n$$
$$S_n = \frac{a_1\left(1 - r^n\right)}{1 - r} \quad r \neq 1.$$

Example 5. Evaluate. $\sum_{n=1}^{20} 2(0.1)^n = \frac{0.2\left(1-0.1^{20}\right)}{1-0.1} = \frac{2}{9}$

III. Geometric Series (p. 667) Pace: 5 minutes

- Discuss the *Exploration* on page 667 of the text.
- State that if $|r| < 1$, then $r^n \to 0$ as $n \to \infty$. Therefore
  $\frac{a_1\left(1-r^n\right)}{1-r} \to \frac{a_1(1-0)}{1-r} = \frac{a_1}{1-r}$ as $n \to \infty$.

- State the following formula for the **sum of an infinite geometric series.**
  $\sum_{n=1}^{\infty} a_1 r^{n-1} = \frac{a_1}{1-r}$

Example 6. Evaluate. $\sum_{n=1}^{\infty} 2(0.4)^{n-1} = \frac{2}{1-0.4} = \frac{10}{3}$

IV. Application (p. 668) Pace: 5 minutes

Example 7. A ball is dropped from a height of 10 feet. Each time it bounces back up, it bounces 0.65 times as high as it did on the previous bounce. What is the total distance traveled by the ball?

$10 + 2\sum_{n=1}^{\infty} 10(0.65)^n = 10 + 2\left(\frac{6.5}{1-0.65}\right) \approx 47.14'$ feet

- Assign the *Writing About Mathematics* on page 668 of the text.