Chapter 9 Sequences, Series, and Probability

Course/Section
Lesson Number
Date

Section 9.5 The Binomial Theorem

Section Objectives: Students will know how to use the Binomial Theorem and Pascal's Triangle to calculate binomial coefficients and binomial expansions.

I. Binomial Coefficients (pp. 683–684) Pace: 10 minutes

- State the **Binomial Theorem.**

$$(x+y)^n = \sum_{r=0}^{n} {}_nC_r x^{n-r} y^r \text{, where } {}_nC_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}.$$

Example 1. Evaluate the following.

a) ${}_{10}C_5 = \frac{10!}{5!5!} = 252$

b) $\binom{8}{2} = \frac{8!}{2!6!} = 28$

c) ${}_{12}C_4 = \frac{12!}{4!8!} = 495$

d) ${}_{12}C_8 = \frac{12!}{8!4!} = 495$

- State the following two facts about binomial coefficients, the first of which was illustrated in parts c) and d) of the above example.
  1. ${}_nC_r = {}_nC_{n-r}$
  2. ${}_nC_n = {}_nC_0 = 1$

II. Pascal's Triangle (p. 685) Pace: 5 minutes

- State that Pascal's Triangle is formed as follows:
  1. The top row is a single 1.
  2. The first and last number in each row is 1.
  3. Every other number in the row is the sum of two numbers immediately above it.
- Pascal noticed that the *n*th row of this triangle consisted of the coefficients of the expansion of $(x+y)^n$.

III. Binomial Expansions (pp. 686–687) Pace: 20 minutes

Example 2. Find the expansion of each of the following expressions.

a)

$$(x+2)^4 = \sum_{r=0}^{4} {}_4C_r x^{4-r} 2^r$$

$$= {}_4C_0 x^{4-0} 2^0 + {}_4C_1 x^{4-1} 2^1 + {}_4C_2 x^{4-2} 2^2 + {}_4C_3 x^{4-3} 2^3 + {}_4C_4 x^{4-4} 2^4$$

$$= x^4 + 8x^3 + 24x^2 + 32x + 16$$

b)

$$(2x+3)^5 = \sum_{r=0}^{5} {}_5C_r (2x)^{5-r} 3^r$$

$$= {}_5C_0 (2x)^5 3^0 + {}_5C_1 (2x)^4 3^1 + {}_5C_2 (2x)^3 3^2$$

$$+ {}_5C_3 (2x)^2 3^3 + {}_5C_4 (2x)^1 3^4 + {}_5C_5 (2x)^0 3^5$$

$$= 32x^5 + 240x^4 + 720x^3 + 1080x^2 + 810x + 243$$

c)

$$(3x-y)^4 = \sum_{r=0}^{4} {}_4C_r (3x)^{4-r} (-y)^r$$

$$= {}_4C_0 (3x)^4 (-y)^0 + {}_4C_1 (3x)^3 (-y)^1$$

$$+ {}_4C_2 (3x)^2 (-y)^2 + {}_4C_3 (3x)^1 (-y)^3 + {}_4C_4 (3x)^0 (-y)^4$$

$$= 81x^4 - 108x^3y + 54x^2y^2 - 12xy^3 + y^4$$

Example 3. Find the coefficient of x^8 in the expansion of $(x^2 + 2)^{12}$
First we need to find r. Since $(x^2)^{12-r} = x^8$, $r = 8$. So, the coefficient of x^8 is ${}_{12}C_8 \cdot 2^8 = 126{,}720$.

Example 4. Find the fifth term of the expansion of $(2x - 3y)^{10}$.
For the fifth term, $r = 4$. So, the fifth term is

$${}_{10}C_4 (2x)^6 (-3y)^4 = 1{,}088{,}640x^6y^4.$$

- Assign the *Writing About Mathematics* on page 687 of the text.