Chapter 1 Functions and Their Graphs

Course/Section Lesson Number Date

Section 1.1 Rectangular Coordinates

Section Objectives: Students will know how to plot points in the coordinate plane and use the Distance and Midpoint Formulas.

I. The Cartesian Plane (pp. 2−3) Pace: 5 minutes

- Draw two real number lines, one horizontal and the other vertical, intersecting at their origins to form the rectangular coordinate system. Label the origin, the x-axis, and the y-axis. Explain that the real number lines divide the plane into four sections called **quadrants** (label these on the coordinate plane). Point out that each point in the plane can be represented by an **ordered pair** (x, y) of real numbers called the **coordinates** of the point. Define the ***x*-coordinate** as the directed distance from the y-axis to the point, and the ***y*-coordinate** as the directed distance from the x-axis to the point.

II. The Pythagorean Theorem and the Distance Formula (pp. 4−5)
Pace: 5 minutes

- Use the Pythagorean Theorem to develop the Distance Formula as follows.

  Plot two arbitrary points and label them (x_1, y_1) and (x_2, y_2). Use the line segment between the two points as the hypotenuse of the right triangle, formed by a horizontal and vertical leg, with these points at the acute vertices. Then,

$$c^2 = a^2 + b^2$$
$$d^2 = |x_2 - x_1|^2 + |y_2 - y_1|^2$$
$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Example 1 Find the distance between (2, –5) and (8, 3).

Let $(x_1, y_1) = (2, -5)$ and $(x_2, y_2) = (8, 3)$.

$$\begin{aligned} d &= \sqrt{(8-2)^2 + (3-(-5))^2} \\ &= \sqrt{6^2 + 8^2} \\ &= \sqrt{36 + 64} \\ &= \sqrt{100} = 10 \end{aligned}$$

Example 2 Show that the points (1, –3), (3, 2), and (–2, 4) form an isosceles triangle.

$$d_1 = \sqrt{(3-1)^2 + (2-(-3))^2} = \sqrt{29}$$
$$d_2 = \sqrt{(-2-3)^2 + (4-2)^2} = \sqrt{29}$$

Since two of its sides have equal length, the triangle is isosceles.

III. The Midpoint Formula (p. 5) Pace: 10 minutes

- Explain that the midpoint of the line segment joining the points (x_1, y_1) and (x_2, y_2) can be determined by the Midpoint Formula.

$$\text{Midpoint} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

- Present the proof of the Midpoint Formula.

Proof
It must be shown that $d_1 = d_2$ and that $d_1 + d_2 = d_3$. (Note that d_1 is the distance from (x_1, y_1) to the midpoint, d_2 is the distance from (x_2, y_2) to the midpoint, and d_3 is the distance from (x_1, y_1) to (x_2, y_2).)

$$d_1 = \sqrt{\left(\frac{x_1 + x_2}{2} - x_1\right)^2 + \left(\frac{y_1 + y_2}{2} - y_1\right)^2}$$
$$= \sqrt{\left(\frac{x_1 + x_2}{2} - \frac{2x_1}{2}\right)^2 + \left(\frac{y_1 + y_2}{2} - \frac{2y_1}{2}\right)^2}$$
$$= \sqrt{\left(\frac{x_2 - x_1}{2}\right)^2 + \left(\frac{y_2 - y_1}{2}\right)^2} = \frac{1}{2}\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$d_2 = \sqrt{\left(x_2 - \frac{x_1 + x_2}{2}\right)^2 + \left(y_2 - \frac{y_1 + y_2}{2}\right)^2}$$
$$= \sqrt{\left(\frac{2x_2}{2} - \frac{x_1 + x_2}{2}\right)^2 + \left(\frac{2y_2}{2} - \frac{y_1 + y_2}{2}\right)^2}$$
$$= \sqrt{\left(\frac{x_2 - x_1}{2}\right)^2 + \left(\frac{y_2 - y_1}{2}\right)^2} = \frac{1}{2}\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

So, $d_1 = d_2$ and

$$d_1 + d_2 = \frac{1}{2}\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} + \frac{1}{2}\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= d_3$$

Tip: Use the figure on page 124 of the text to help demonstrate the proof.

Example 3 Find the midpoint of the line segment joining the points (–9, 5) and (4, 2).

$$\left(\frac{-9+4}{2}, \frac{5+2}{2}\right) = \left(-\frac{5}{2}, \frac{7}{2}\right)$$

IV. Applications (pp. 6−8) Pace: 10 minutes

Review the geometry formulas at the bottom of page 7 of the text. Assign the *Writing About Mathematics* activity at the bottom of page 8 of the text.