Chapter 1 Functions and Their Graphs

Course/Section Lesson Number Date

Section 1.10 Mathematical Modeling and Variation

Section Objectives: Students will know how to write mathematical models for direct, inverse, and joint variation. In addition, students will know how to use the least squares regression feature of a graphing utility to find mathematical models for actual data.

I. Introduction (p. 103) Pace: 5 minutes

- State that in Section 1.3 we learned how to fit a linear equation to two data points. In this section we expand on this idea.

Example 1. The chart below gives the profit for a company for the years 1990 to 1999, where 0 corresponds to 1990 and the profit is in millions of dollars.

Year	0	1	2	3	4	5	6	7	8	9
Profit	5.1	5.22	5.44	5.56	5.8	5.99	6.22	6.68	6.6	6.77

The company feels that the data can modeled by

$y = 0.2x + 5$

Make a scatter plot of the data and graph the line. Is this a good model for the data? Yes

II. Least Squares Regression and Graphing Utilities (p. 104)
Pace: 5 minutes

- Discuss the **least squares regression line.** Say that you have a set of data points and you want the best-fitting line for the data. The **deviation** is the difference between the y-value of a point and the corresponding y-value of the line of best fit. Statisticians then minimize the sum of the squared deviations.
- When you run a linear regression program, the "r-value" or **correlation coefficient** gives a measure of how well the model fits the data. The closer $|r|$ is to 1, the better the fit.

Example 2. Take the data from Example 1 above and use a graphing utility to find the least squares regression line for the data.

$y = 0.2x + 5.036$

III. Direct Variation (p. 105) Pace: 5 minutes

- State the definition of **direct variation** by stating the following equivalent statements:
  1. y **varies directly** as x.
  2. y is **directly proportional** to x.
  3. $y = kx$ for some nonzero constant k, called the **constant of variation** or the **constant of proportionality.**

Example 3. y varies directly as x. $y = 15$ when $x = 3$. Find a mathematical model that gives y in terms of x.

$$y = kx$$
$$15 = k \cdot 3$$
$$5 = k$$
$$y = 5x$$

IV. Direct Variation as an *n*th Power (p. 106) Pace: 5 minutes

- State the definition of **direct variation as an *n*th power** by stating the following equivalent statements:
  1. y **varies directly as the *n*th power** of x.
  2. y is **directly proportional to the *n*th power** of x.
  3. $y = kx^n$ for some nonzero constant k.

  Example 4. The area of a circle is directly proportional to the square of its diameter. Find a mathematical model that gives the area of a circle in terms of its diameter if the area is 16π when the diameter is 8.

$$A = kd^2$$
$$16\pi = k \cdot 8^2$$
$$16\pi = k \cdot 64$$
$$\frac{\pi}{4} = k$$
$$A = \frac{\pi}{4} d^2$$

V. Inverse Variation (p. 107) Pace: 5 minutes

- State the definition of **inverse variation** by stating the following equivalent statements:
  1. y **varies inversely as the *n*th power** of x.
  2. y is **inversely proportional to the *n*th power** of x.
  3. $y = k/x^n$ for some nonzero constant k.

  Example 5. y varies inversely as the cube of x. $y = 54$ when $x = 3$. Find y when $x = 2$.

$$y = k / x^3$$
$$54 = k / 3^3$$
$$54 = k / 27$$
$$1458 = k$$
$$y = 1548 / x^3$$
$$y = 1548 / \left(2^3\right) = 182.25$$

VI. Joint Variation (p. 108) Pace: 5 minutes

- State the definition of **joint variation** by stating the following equivalent statements:
  1. z **varies jointly** as the nth power of x and the mth power of y.
  2. z is **jointly proportional to** the nth power of x and the mth power of y.
  3. $y = kx^n y^m$ for some nonzero constant k.

  Example 6. The volume of a right circular cylinder is jointly proportional to its height and to the square of its diameter. If the volume is 320π cm^3 when the diameter is 16 cm and the height is 5 cm, what is the volume when the diameter is 10 cm and the height is 4 cm?

$$V = kd^2h$$
$$320\pi = k \cdot 16^2 \cdot 5$$
$$320\pi = 1280k$$
$$\frac{\pi}{4} = k$$
$$V = \frac{\pi}{4} \cdot 10^2 \cdot 4 = 100\pi$$