Chapter 1 Functions and Their Graphs

Course/Section
Lesson Number
Date

Section 1.2 Graphs of Equations

Section Objectives: Students will know how to sketch the graph of an equation.

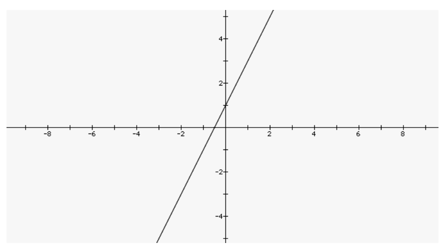
I. The Graph of an Equation (pp. 14–17) Pace: 10 minutes

- A solution of an equation in two variables x and y is an ordered pair (a, b) such that when x is replaced by a and y is replaced by b, the resulting equation is a true statement. The graph of an equation of this type is the collection of all points in the rectangular coordinate system that correspond to the solution of the equation.

Example 1. Sketch the graph of the following.

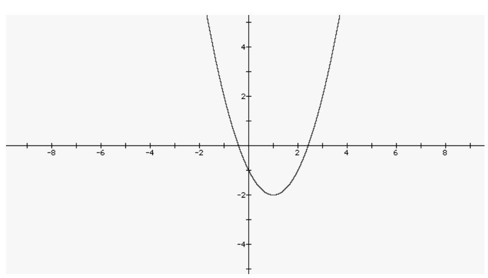
a) $y = 2x + 1$

x	-2	-1	0	1	2
$y = 2x + 1$	-3	-1	1	3	5
(x, y)	(-2, -3)	(-1, -1)	(0, 1)	(1, 3)	(2, 5)



b) $y = x^2 - 2x - 1$

x	-3	-2	-1	0	1	2
$y = x^2 - 2x - 1$	14	7	2	-1	-2	-1
(x, y)	(-3, 14)	(-2, 7)	(-1, 2)	(0, -1)	(1, -2)	(2, -1)

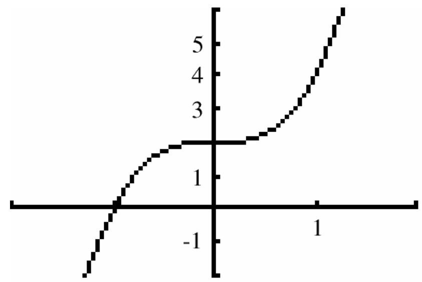


- Note that the point-plotting method is easy, but as our equations get more complicated, we will need to use other methods to graph them.

II. Intercepts of a Graph (p. 17) Pace: 10 minutes

- A point at which the graph of an equation meets the x-axis is called an ***x*-intercept.** A point at which the graph of an equation meets the y-axis is called a ***y*-intercept.** It is possible for a graph to have no intercepts, one intercept, or several intercepts.

Example 2. Find the x- and y-intercepts of the graph of $y = 2x^3 + 2$ shown below.

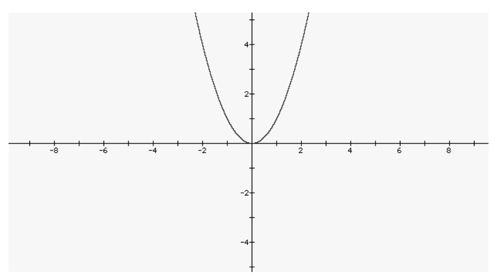


You can see that the graph of $y = 2x^3 + 2$ has an x-intercept (where y is 0) at $(-1, 0)$ and a y-intercept (where x is zero) at $(0, 2)$.

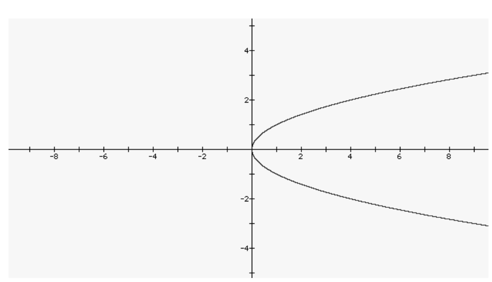
III. Symmetry (pp. 18−20) Pace: 15 minutes

Graphical Tests for Symmetry

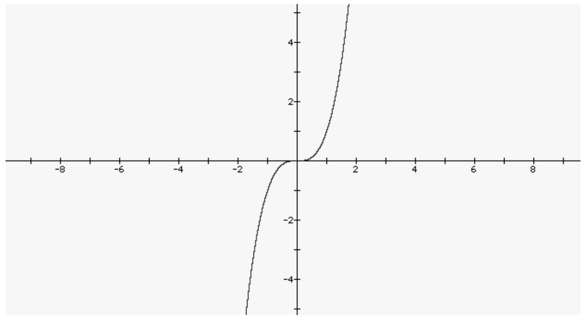
- A graph is **symmetric with respect to the y-axis** if, whenever (x, y) is on the graph, $(-x, y)$ is also on the graph. As an illustration of this, we graph $y = x^2$.



- A graph is **symmetric with respect to the x-axis** if, whenever (x, y) is on the graph, $(x, -y)$ is also on the graph. As an illustration of this, we graph $y^2 = x$.



- A graph is **symmetric with respect to the origin** if, whenever (x, y) is on the graph, ($-x$, $-y$) is also on the graph. As an illustration of this, we graph $y = x^3$.



Algebraic Tests for Symmetry

- The algebraic tests for symmetry are as follows:
  - (i) The graph of an equation is symmetric with respect to the y-axis if replacing x with $-x$ yields an equivalent equation.
  - (ii) The graph of an equation is symmetric with respect to the x-axis if replacing y with $-y$ yields an equivalent equation.
  - (iii) The graph of an equation is symmetric with respect to the origin if replacing x with $-x$ and y with $-y$ yields an equivalent equation.

Example 3. The graph of $y = x^3 - x$ is symmetric with respect to the origin because

$$y = x^3 - x$$
$$-y = (-x)^3 - (-x)$$
$$-y = -x^3 + x$$
$$y = x^3 - x$$

IV. Circles (p. 20) Pace: 5 minutes

- A circle with center at (h, k) and radius r consists of all points (x, y) whose distance from (h, k) is r. From the Distance Formula, we have $\sqrt{(x-h)^2 + (y-k)^2} = r \Rightarrow (x-h)^2 + (y-k)^2 = r^2$ as the standard form equation of a circle.

Example 4. Find the standard form of the equation of the circle with center at (2, −5) and radius 4.

$(x-2)^2 + (y-(-5))^2 = 4^2$

or

$(x-2)^2 + (y+5)^2 = 16$

V. Application (p. 21) Pace: 10 minutes

Review Example 9 on page 21 of the text. Emphasize the three approaches to problem-solving: numerical (construct a table), graphical (draw a graph), and algebraic (use the rules of algebra to confirm the graphical solution).