Chapter 1 Functions and Their Graphs

Course/Section
Lesson Number
Date

Section 1.4 Functions

Section Objectives: Students will know how to use function notation and how to evaluate functions and find their domains.

I. Introduction to Functions (pp. 40−42) Pace: 10 minutes

- Define a **relation** to be a set of ordered pairs. The **domain** of a relation is the set of all x-coordinates of the relation. The **range** of a relation is the set of all y-coordinates of the relation. A **function** is a relation in which each domain element corresponds to one and only one range element.

Example 1. $S = \{(1, 3), (-5, 4), (7, 3)\}$

a) What is the domain of S?
$\{1, -5, 7\}$

b) What is the range of S?
$\{3, 4\}$

c) Is S a function?
Yes. Several students will say no, because of the (1, 3) and (7, 3). Show them that this does not violate the definition of function.

- State that since the solution set of an equation in two variables is a set of ordered pairs, such equations define relations. In addition, some will define functions. In this case, the variable representing the domain elements (x) is called the **independent variable.** The variable representing the range elements (y) is called the **dependent variable.** An equation in two variables x and y represents y as a function of x if it can be uniquely solved for x.

Example 2. Which of the following equations represent y as a function of x?

a) $2x^2 + y + 1 = 0$

$$y = -2x^2 - 1$$

The equation is uniquely solved for y; hence y is a function of x.

b)

$$x + y^2 - 6 = 0$$
$$y^2 = 6 - x$$
$$y = \pm\sqrt{6 - x}$$

The equation is not uniquely solved for y; hence y is not a function of x.

II. Function Notation (pp. 42−43) Pace: 10 minutes

- Discuss *naming* a function so that it can be referenced. Name the function f. Since we say y is a function of x, replace y with $f(x)$. Now, if we need to know the value of y in the function f when x equals a, we just have to write $f(a)$. This value is read "f of a."

Tip: There are two concepts that you cannot emphasize too much. One is that $y = f(x)$. The other is that f is the *name* of the function, not a variable.

Example 3. Given $f(x) = x^2 - 4x$, find the following.

a)

$$f(3) = 3^2 - 4 \cdot 3$$
$$= 9 - 12$$
$$= -3$$

b) $f(a) = a^2 - 4a$

c) $f(x+h) = (x+h)^2 - 4(x+h)$

$$= x^2 + 2xh + h^2 - 4x - 4h$$

Example 4. Given $f(x) = \begin{cases} x+1, & \text{if } x \ge 0 \\ -x, & \text{if } x < 0 \end{cases}$, find the following.

a) $f(2) = 2 + 1 = 3$, since $2 \ge 0$.

b) $f(-5) = -(-5) = 5$, since $-5 < 0$.

III. The Domain of a Function (p. 44) Pace: 10 minutes

- Define the **implied domain** of a function to be the set of all x such that the corresponding y is a real number. At least for a while, we will consider only three situations:
  1. Polynomials $\Rightarrow$ domain is $(-\infty, \infty)$.
  2. Fractions $\Rightarrow$ cannot be any number in the domain that makes the denominator zero.
  3. Radicals $\Rightarrow$ if the index is even, then the radicand must be nonnegative.

Example 5. Find the domains of the following functions.

a) $f(x) = x^3 + 3x + 1$

This is a polynomial, so the domain is $(-\infty, \infty)$.

b) $f(x) = \dfrac{2}{x+2}$

$x + 2 \ne 0$, so the domain is $(-\infty, -2) \cup (-2, \infty)$.

c) $f(x) = \sqrt[4]{5-x}$

$5 - x \ge 0$, so the domain is $(-\infty, 5]$.

IV. Applications (pp. 45−46) Pace: 10 minutes

Example 6. A baseball is hit at a point 3 feet above the ground at a velocity of 100 feet per second and at an angle of 45 degrees. The path of the baseball is given by the equation

$$f(x) = -0.0032x^2 + x + 3$$

where x and y are measured in feet. Will the baseball clear a 10-foot fence located 315 feet from home plate?

When $x = 315$, the height of the baseball is

$$f(315) = -0.0032(315)^2 + 315 + 3$$
$$= 0.48 \text{ feet.}$$

So, the baseball will not clear the fence.

V. Difference Quotients (pp. 46–47) Pace: 10 minutes

- We will now work with an expression from calculus called the **difference quotient.**

Example 7. Given $f(x) = x^2 + x - 1$, find $\dfrac{f(x+h) - f(x)}{h}$.

$$\begin{aligned} \frac{f(x+h) - f(x)}{h} &= \frac{\left[(x+h)^2 + (x+h) - 1\right] - \left[x^2 + x - 1\right]}{h} \\ &= \frac{x^2 + 2xh + h^2 + x + h - 1 - x^2 - x + 1}{h} \\ &= \frac{2xh + h^2 + h}{h} \\ &= 2x + h + 1 \\ &\quad (h \neq 0) \end{aligned}$$

Tip: The majority of your students will start to have real trouble here. You may need several more examples like this.