Chapter 1 Functions and Their Graphs

Course/Section
Lesson Number
Date

Section 1.5 Analyzing Graphs of Functions

Section Objectives: Students will know how to use the Vertical Line Test, find zeros of functions, identify intervals on which functions are increasing or decreasing, determine average rate of change, and identify even and odd functions.

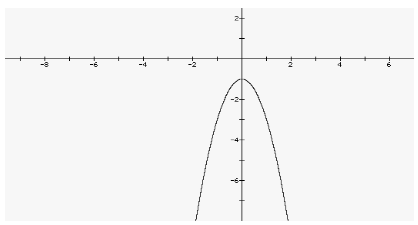
I. The Graph of a Function (pp. 54–55) Pace: 10 minutes

- Tell students that because $y = f(x)$, graphing functions is no different from graphing equations in two variables (Section 1.2).
- Draw the graph of $y^2 = x$ and ask the class if it is the graph of a function. This can lead to a discussion of the **Vertical Line Test,** which states:

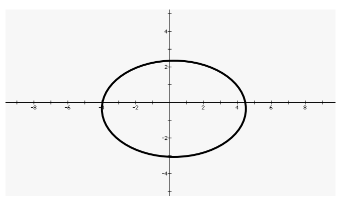
  If every vertical line meets the graph of a relation in at most one point, then the relation is a function.

Example 1. Which of the following graphs are graphs of functions?

a) Yes



b) No



II. Zeros of a Function (p. 56) Pace: 5 minutes

- State that if the graph of a function of x has an x-intercept at $(a, 0)$, then a is a **zero** of the function. That is, $f(a) = 0$.

Tip: Graph each of the functions in the following example using a graphing utility.

Example 2. Find the zeros of the following functions.

a) $f(x) = 2x^2 - x - 1$

$$2x^2 - x - 1 = 0$$

$$(2x + 1)(x - 1) = 0$$

$$2x + 1 = 0 \Rightarrow x = -\frac{1}{2}$$

$$x - 1 = 0 \Rightarrow x = 1$$ The zeros are –1/2 and 1.

b) $g(x) = \sqrt[3]{x-1}$

$$\sqrt[3]{x-1} = 0$$

$$x - 1 = 0$$

$$x = 1$$ The zero is 1.

c) $h(x) = \frac{x-5}{2x-1}$

$$\frac{x-5}{2x-1} = 0$$

$$x - 5 = 0$$

$$x = 5$$

The zero is 5.

III. Increasing and Decreasing Functions (pp. 57−58)
Pace: 10 minutes

- Draw the graph of any polynomial, such as $f(x) = x^3 + 3x^2$. Label the extrema at –2 and 0. Ask the class to identify any intervals over which they feel the function is increasing. Do the same for intervals over which the function is decreasing. Now tell the students that they have the intuition to understand the following definition.
    1. A function f is **increasing** on an interval if, for any x_1 and x_2 in the interval such that $x_1 < x_2$, $f(x_1) < f(x_2)$.
    2. A function f is **decreasing** on an interval if, for any x_1 and x_2 in the interval such that $x_1 < x_2$, $f(x_1) > f(x_2)$.
    3. A function f is **constant** on an interval if, for any x_1 and x_2 in the interval, $f(x_1) = f(x_2)$.
- State the following definition.

  $f(a)$ is a **relative minimum** of the function f if there exists an open interval I, containing a, such that $f(a) \le f(x)$ for all x in I. $f(a)$ is a **relative maximum** of the function f if there exists an open interval I, containing a, such that $f(a) \ge f(x)$ for all x in I.

  Example 3. Use a graphing utility to estimate the relative maximum of $f(x) = -3x^2 - 2x + 1$.
  Graph the function and use the maximum command to find the relative maximum, which is (−0.33, 1.33).

IV. Average Rate of Change (p. 59) Pace: 5 minutes

- Remind students that in Section 1.3, they learned that the slope of a line can be thought of as a *rate of change*—the change in y (vertical change) over the change in x (horizontal change). For a nonlinear graph, the slope changes at each point on the graph. The rate of change between any two points on a nonlinear graph is called the **average rate of change.** It is the slope of the line through the two chosen points, called the **secant line.**

  Example 4. Find the average rate of change of $f(x) = x^2 - 2x + 3$ from $x_1 = 0$ to $x_2 = 4$.
  The average rate of change is
  $$\frac{f(x_2) - f(x_1)}{x_2 - x_1} = \frac{f(4) - f(0)}{4 - 0} = \frac{11}{4}$$
  Because the average rate of change is positive, the secant line has a positive slope.

V. Even and Odd Functions (p. 60) Pace: 10 minutes

- State that **even functions** are symmetric with respect to the y-axis and **odd functions** are symmetric with respect to the origin. Then state the following test.
    1. A function f is even if, for each x in the domain of f, $f(-x) = f(x)$.
    2. A function f is odd if, for each x in the domain of f, $f(-x) = -f(x)$.

Tip: Remind the students that whether they are testing for odd or even, they should begin the same way, with $f(-x)$.

Example 5. Determine if the following functions are odd or even.

a)

$$f(x) = x^4 - |x|$$
$$f(-x) = (-x)^4 - |-x|$$
$$= x^4 - |x|$$
$$= f(x)$$

Therefore, the function is even.

b)

$$g(x) = \frac{x}{x^2 + 1}$$
$$g(-x) = \frac{-x}{(-x)^2 + 1}$$
$$= -\frac{x}{x^2 + 1}$$
$$= -g(x)$$

Therefore, the function is odd.

c)

$$h(x) = x + 6$$
$$h(-x) = -x + 6$$
$$\neq h(x), \text{ nor } -h(x)$$

Therefore, the function is neither even nor odd.