Chapter 1 Functions and Their Graphs

Course/Section
Lesson Number
Date

Section 1.6 A Library of Parent Functions

Section Objectives: Students will know how to identify and graph linear, squaring, cubic, square root, reciprocal, step, and piecewise-defined functions, and how to recognize the graphs of common functions.

I. Linear and Squaring Functions (pp. 66−67) Pace: 10 minutes

Tell students that the graph of every **linear function** $f(x) = ax + b$ is a line with slope $m = a$ and y-intercept $(0, b)$. The graph of a linear function has the following features:

- Domain: All real numbers
- Range: All real numbers
- One intercept at $(0, b)$.
- The graph is increasing if $m > 0$, decreasing if $m < 0$, and constant if $m = 0$.

Example 1. Write the linear function f for which $f(2) = 4$ and $f(5) = -1$.

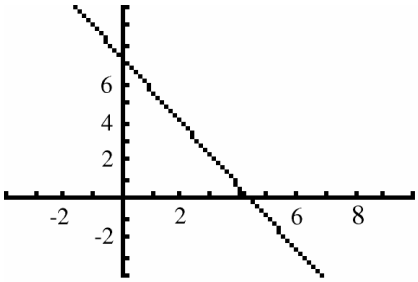
First we find the slope and then we use point-slope form.

$$m = \frac{-1-4}{5-2} = -\frac{5}{3}$$

$$y - 4 = -\frac{5}{3}(x-2)$$

$$y = -\frac{5}{3}x + \frac{22}{3}$$

$$f(x) = -\frac{5}{3}x + \frac{22}{3}$$



- Tell students there are two special linear functions. A **constant function** has the form $f(x) = c$. The graph of a constant function is a horizontal line. The **identity function** has the form $f(x) = x$. The graph of the identity function has slope 1 and passes through (0, 0). For the identity function, every x-coordinate is the same as its corresponding y-coordinate.

- The graph of the **squaring function** $f(x) = x^2$ is a U-shaped curve. The graph has the following features:

  - Domain: All real numbers
  - Range: All *nonnegative* real numbers
  - Intercept at (0, 0)
  - Decreasing on $(-\infty, 0)$ and increasing on $(0, \infty)$
  - Symmetric with respect to the y-axis
  - Relative minimum at (0, 0)

II. Cubic, Square Root, and Reciprocal Functions (p. 68)
Pace: 10 minutes

- Refer to page 68 of the text for the features of cubic, square root, and reciprocal functions.

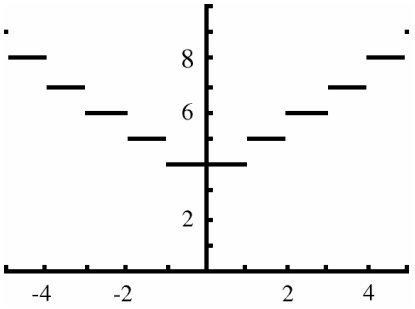
III. Step and Piecewise-Defined Functions (pp. 69−70)
Pace: 10 minutes

- Define the greatest integer function, denoted by $[|x|]$, to be
  $[|x|]$ = the greatest integer less than or equal to x.
  This is commonly referred to as a **step function**. It is the type of function telephone companies use to bill us for long distance calls. Graph this function with a graphing utility in *dot mode*, not connected mode.

  Example 2. Evaluate the function when $x = -2$, 5/2, and 4.
  $f(x) = [|x|] + 4$

  For $x = -2$, the greatest integer less than or equal to –2 is –2, so
  $f(x) = [|-2|] + 4 = -2 + 4 = 2$.
  For $x = 5/2$, the greatest integer less than or equal to 5/2 is 2, so
  $f(x) = [|5/2|] + 4 = 2 + 4 = 6$.
  For $x = 4$, the greatest integer less than or equal to 4 is 4, so
  $f(x) = [|4|] + 4 = 4 + 4 = 8$.

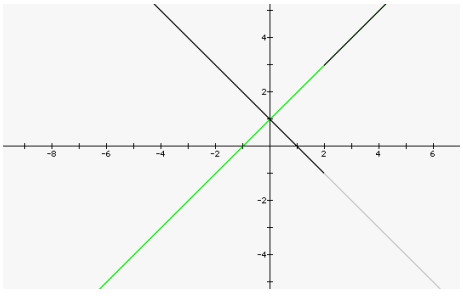
  The graph is shown below.



- Discuss how to graph piecewise-defined functions by drawing the whole graph for one part of the function on the board, then erasing the part that you do not want.

  Example 3. Sketch the graph of $f(x) = \begin{cases} x+1, & \text{if } x \le 2 \\ 1-x, & \text{if } x > 2 \end{cases}$

  Draw, then erase the light parts.



IV. Parent Functions (p. 70) Pace: 5 minutes

- State that the eight most common functions are:
  - The constant function, $f(x) = c$
  - The identity function, $f(x) = x$
  - The absolute value function, $f(x) = |x|$
  - The square root function, $f(x) = \sqrt{x}$
  - The quadratic function, $f(x) = x^2$
  - The cubic function, , $f(x) = x^3$
  - The reciprocal function, $f(x) = 1/x$
  - The greatest integer function, $f(x) = [|x|]$

- Inform the students that they need to be very familiar with the graphs of the above functions. You should draw a quick (two seconds) sketch of each and tell the students that they should be able to draw them just as quickly. The graphs of all of these functions are found on page 70 of the text. Students should take the time to study the graphs carefully and commit them to memory.