Chapter 1 Functions and Their Graphs

Course/Section
Lesson Number
Date

Section 1.7 Transformations of Functions

Section Objectives: Students will know how to identify and graph shifts, reflections, and nonrigid transformations of functions.

I. Shifting Graphs (pp. 74−75) Pace: 10 minutes

- Use a graphing utility to graph $Y_1 = f(x) = x^2$. Then, on the same viewing screen, graph $Y_2 = (x - 4)^2$. Note that this is $f(x - 4)$. Repeat for $Y_3 = (x + 4)^2$, $Y_4 = x^2 + 4$, and $Y_5 = x^2 - 4$. Next, state the following facts.

  Let c be a positive real number. The following changes in the function $y = f(x)$ will produce the stated shifts in the graph of $y = f(x)$.
  1. $h(x) = f(x - c)$ Horizontal shift c units to the right
  2. $h(x) = f(x + c)$ Horizontal shift c units to the left
  3. $h(x) = f(x) - c$ Vertical shift c units downward
  4. $h(x) = f(x) + c$ Vertical shift c units upward

Example 1. Given $f(x) = x^3 + x$, describe the shifts of the graph of f generated by the following functions.

a) $g(x) = (x + 1)^3 + x + 1$ Horizontal shift 1 unit to the left.

b) $h(x) = (x - 4)^3 + x - 4$ Horizontal shift 4 units to the right.

II. Reflecting Graphs (pp. 76−77) Pace: 10 minutes

- Use a graphing utility to graph $Y_1 = f(x) = (x - 2)^3$. Then, on the same viewing screen, graph $Y_2 = -(x - 2)^3$. Note that this is $-f(x)$. Repeat for $Y_3 = (-x - 2)^3$. Next, state the following facts.

  The following changes in the function $y = f(x)$ will produce the stated reflections in the graph of $y = f(x)$.
  1. $h(x) = -f(x)$: reflection in the x-axis
  2. $h(x) = f(-x)$: reflection in the y-axis

Example 2. Given $f(x) = x^3 + 3$, describe the reflections of the graph of f generated by the following functions.

a) $g(x) = (-x)^3 + 3$ Reflection in the y-axis.

b) $h(x) = -x^3 - 3$ Reflection in the x-axis.

III. Nonrigid Transformations (p. 78) Pace: 5 minutes

- Nonrigid transformations actually *distort* the shape of a graph, instead of just shifting or reflecting it. One nonrigid transformation of $y = f(x)$ comes from equations of the form $g(x) = cf(x)$. If $c > 1$, there is a **vertical stretch** of the graph of $y = f(x)$. If $0 < c < 1$, there is a **vertical shrink** of $y = f(x)$.
- Another nonrigid transformation of $y = f(x)$ comes from equations of the form $h(x) = f(cx)$. If $c > 1$, there is a **horizontal shrink** of the graph of $y = f(x)$. If $0 < c < 1$, there is a **horizontal stretch** of $y = f(x)$.

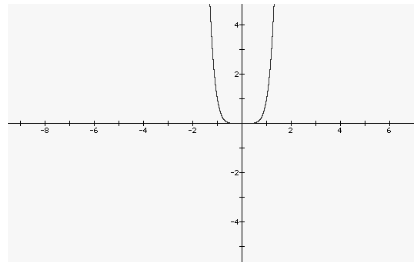
Example 3. Compare the graph of each function with the graph of $f(x) = 4 + x^2$.

(a) $g(x) = f(2x)$

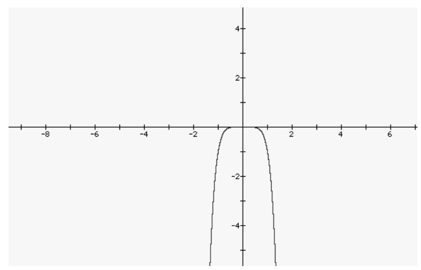
Relative to the graph of $f(x)$, the graph of
$g(x) = f(2x) = 4 + (2x)^2 = 4 + 4x^2$
is a horizontal shrink (each x-value is multiplied by ½) of the graph of f.

(b) $h(x) = f(1/3x)$
Relative to the graph of $f(x)$, the graph of
$h(x) = f(1/3x) = 4 + (1/3x)^2 = 4 + 1/9x^2$
is a horizontal stretch (each x-value is multiplied by 3) of the graph of f.

Example 4. Below is the graph of $y = f(x)$.



a) Graph $y = -f(x)$.



b) Graph $y = f(-x) + 1$